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THE UNIRATIONALITY OF CERTAIN ELLIPTIC SURFACES IN CHARACTERISTIC p

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0. Introduction. Let X be a non-singular projective surface defined over an algebraically closed field k of characteristic $p \geqq 5$. X is called unirational if there exists a generically surjective rational mapping from the projective plane $\boldsymbol{P}^2$ to S. Since Zariski found an example of irrational unirational algebraic surfaces in positive characteristics (cf. Zariski [26]), many such surfaces were found and investigated by Artin (cf. [1]), Shioda (cf. [21], [22], [23] and [24]), Miyanishi (cf. [9], [10] and [11]), Rudakov and Shafarevich (cf. [16]), Blass (cf. [2]) and others. In the previous note [6], we introduced the notion of a unirational elliptic surface of base change type, and characterized irrational unirational elliptic surfaces of base change type with sections. In that case, we considered two classes of elliptic surfaces defined by the following equations:

$$\text{(I)} \qquad y^2 = 4x^3 - t^3(t-1)^3(t-\alpha)^3x\,,$$

$$\text{(II)} \qquad y^2 = 4x^3 - t^5(t-1)^5(t-\alpha)^5(t-\beta)^5(t-\gamma)^5\,,$$

where t is a local coordinate of an affine line in $\boldsymbol{P}^1$, and α, β, γ are arbitrary elements of the field k. We proved the following theorem.

THEOREM I. *The elliptic surface defined by the equation* (I) *is unirational if and only if* $p \equiv 3 \bmod 4$.

Moreover, we proved that if $p \equiv 2 \bmod 3$, then the elliptic surface defined by the equation (II) is unirational. In this note, we prove the following theorem which we conjectured in [4].

THEOREM II. *The elliptic surface defined by the equation* (II) *is unirational if and only if* $p \equiv 2 \bmod 3$.

Next, we denote by E_t the elliptic curve over $k(t)$ defined by the equation (I) (resp. (II)), and s the number of points where E_t has bad reduction. We denote by $r(E_t)$ the rank of the abelian group of rational points over $k(t)$ of E_t. Using these notations, as a corollary to Theorems I

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