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THE IRREDUCIBILITY OF AN AFFINE HOMOGENEOUS CONVEX DOMAIN

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Introduction. An affine homogeneous convex domain Ω in the n-dimensional real number space $\boldsymbol{R}^n$ is said to be *reducible* if it is affinely equivalent to a direct product of affine homogeneous convex domains. Otherwise, it is said to be *irreducible*. By using the characteristic function φ_Ω of Ω, we have a Riemannian metric $g_\Omega =$ Hessian of $\log \varphi_\Omega$ on Ω, which is called the *canonical metric* of Ω. The canonical metric is invariant under the group $G(\Omega)$ of all affine automorphisms of Ω (cf. [18], [16]).

With respect to the canonical metric, an affine homogeneous convex domain is a reducible homogeneous Riemannian manifold if it is a reducible convex domain ([16]). It is natural to raise the question whether the irreducibility of a convex domain Ω implies that of the Riemannian manifold (Ω, g_Ω) or not. A homogeneous convex cone is a special case of an affine homogeneous convex domain. It is known that a homogeneous convex cone in $\boldsymbol{R}^n (n \geqq 2)$ is always reducible as a Riemannian manifold ([7], [15]). However, for affine homogeneous convex domains other than homogeneous convex cones, the answer is affirmative. The main purpose of the present paper is to prove this fact. After reviewing results of [18] in §1 and preparing some lemmas in §2, we will prove the main result in §3 (Theorem 3.1).

In §4, we will study Riemannian geometric relations between an affine homogeneous convex domain Ω and the tube domain $D(\Omega)$ over it. It is known that the canonical metric of Ω coincides with the metric induced from the Bergman metric of $D(\Omega)$ (cf. [6]). By using this and a result of [3], we will prove that a tube domain $D(\Omega)$ is irreducible with respect to the Bergman metric if and only if Ω is an irreducible convex domain (Theorem 4.4).

The Bergman metric of an arbitrary homogeneous bounded domain in a complex number space is Einstein (cf. e.g., [5], [12]). In the case of affine homogeneous convex domains, an elementary domain is the only irreducible domain whose canonical metric is Einstein. This fact will be proved in §5 (Theorem 5.1).