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# TSUCHIHASHI'S CUSP SINGULARITIES ARE BUCHSBAUM SINGULARITIES

MASA-NORI ISHIDA

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**Introduction.** In his paper [T], Tsuchihashi studied a kind of "elliptic" isolated singularities which are generalizations, in higher dimensions, of the two-dimensional cusp singularities in the sense of Karras [K] and Nakamura [N1]. However his cusp singularity is not Cohen-Macaulay. He showed that some of his cusp singularities of higher dimensions are quasi-Buchsbaum. This fact suggests that all his cusp singularities are Buchsbaum. Recall that a noetherian local ring $A$ of dimension $d$ is said to be Buchsbaum if there exists an integer $k$ such that, for any system of parameters $\{u_1, \cdots, u_d\}$ of $A$, the difference of the colength length$(A/I)$ and the multiplicity $e_A(I)$ of the ideal $I = (u_1, \cdots, u_d)$ is equal to $k$. It is well known that $A$ is Cohen-Macaulay if and only if it is Buchsbaum and the above constant $k$ is equal to 0. Buchsbaum local rings are quasi-Buchsbaum. But the converse is not true. Actually, S. Goto showed that there are many examples of non-Buchsbaum quasi-Buchsbaum local domains [G].

The purpose of this paper is to prove that Tsuchihashi's cusp singularities are always Buchsbaum. For the proof, we use Schenzel's characterization of Buchsbaum local rings in terms of Grothendieck's dualizing complexes [S]. Actually, we determine the truncation below $\tau_{-1}(K_V^{\cdot})$ of the dualizing complex $K_V^{\cdot}$ of Tsuchihashi's cusp singularity $P \in V$ of dimension $n$.

This problem was raised by Dr. Kimio Watanabe and Mr. Y. Koyama as is mensioned in the introduction of [T].

**1. Preparation.** Let $N$ be a free $\boldsymbol{Z}$-module of rank $n > 1$. Tsuchihashi considers a pair $(C, \Gamma)$ of an open convex cone in $N_{\boldsymbol{R}} = N \otimes_{\boldsymbol{Z}} \boldsymbol{R}$ which contains no lines in $N_{\boldsymbol{R}}$ and a subgroup $\Gamma$ of the automorphism group $\mathrm{GL}(N)$ of $N$ such that $C$ is $\Gamma$-invariant, the action of $\Gamma$ on $D = C/\boldsymbol{R}_+$ is properly discontinuous and free, and has the compact quotient $D/\Gamma$.

There exists a rational partial polyhedral decomposition $\Sigma$ of $N_{\boldsymbol{R}}$ such that

(1) $\bigcup_{\sigma \in \Sigma \setminus \{0\}} (\sigma \setminus \{0\}) = C$,