Tôhoku Math. Journ.
36 (1984), 469–482.

THE FUNCTIONAL EQUATION OF ZETA DISTRIBUTIONS ASSOCIATED WITH FORMALLY REAL JORDAN ALGEBRAS

Dedicated to Prof. M. Koecher on his sixtieth birthday

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(Received February 17, 1984)

The purpose of this paper is to give an explicit expression for the Fourier transform of the zeta distributions on a certain class of prehomogeneous spaces defined by Jordan algebras.

Let V be a formally real simple Jordan algebra over $\boldsymbol{R}$. Let $\dim V = n$ and $\mathrm{rk}\, V = r$ (for definition, see 1.1). We fix a (positive definite) inner product on V defined by

$$(1) \qquad \langle x, y \rangle = \frac{r}{n} \mathrm{tr}(T_{xy}) \quad (x, y \in V) ,$$

where T_x denotes the linear transformation of V defined by $T_x(y) = xy$. The "structure group" of V, $G = \mathrm{Str}(V)$ (see 1.2), is then self-adjoint with respect to $\langle\ \rangle$, and hence is a reductive algebraic group. It is well-known that the pair (G, V) is a (real) prehomogeneous vector space in the sense of Sato-Shintani [6], i.e. if one denotes by G_C and V_C the complexifications of G and V, respectively, G_C is transitive on the Zariski-open set

$$V_C^\times = \{x \in V_C \mid N(x) \neq 0\}$$

(see [5c]). Here N denotes the "reduced norm" of V, which is an absolutely irreducible homogeneous polynomial function on V of degree r, characterized by the property:

$$(2) \qquad N(1) = 1 , \qquad N(gx) = \det(g)^{r/n} N(x) \quad (g \in G^\circ,\ x \in V) ,$$

where G° is the identity connected component of G.

The set of real invertible elements $V^\times = V \cap V_C^\times$ is decomposed into the disjoint union of $r + 1$ (open) G°-orbits:

$$V^\times = \coprod_{i=0}^{r} \Omega_i ,$$

where Ω_i is the set of elements of signature $(r - i, i)$ ([5c]). In particular,