Tôhoku Math. Journ.
36 (1984), 439–452.

ON THE STARK-SHINTANI CONJECTURE AND CYCLOTOMIC Z_p-EXTENSIONS OF CLASS FIELDS OVER REAL QUADRATIC FIELDS II

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(Received October 5, 1983)

Introduction. Let p be a prime number, and denote by $\boldsymbol{Z}_p$ the ring of p-adic integers. In our previous paper [9], we have constructed certain cyclotomic $\boldsymbol{Z}_p$-extensions $M_\infty = \bigcup_{n\geqq 0} M_n$ such that the Stark-Shintani invariants for M_n are units of M_n for each $n \geqq 0$. In this paper, we study the image of these units in the completion of M_∞ at a prime over p.

Let F be a real quadratic field embedded in the real number field $\boldsymbol{R}$. Let M be a finite abelian extension of F in which exactly one of the two infinite primes of F, corresponding to the prescribed embedding of F into $\boldsymbol{R}$, splits. Let $\mathfrak{f}$ be the conductor of M/F. Denote by $H_F(\mathfrak{f})$ the group consisting of all narrow ray classes of F defined modulo $\mathfrak{f}$. Let G be the subgroup of $H_F(\mathfrak{f})$ corresponding to M by class field theory. Take a totally positive integer ν of F satisfying $\nu + 1 \in \mathfrak{f}$, and denote by the same letter ν the narrow ray class modulo $\mathfrak{f}$ represented by the principal ideal (ν). For each $c \in H_F(\mathfrak{f})$, set $\zeta_F(s, c) = \sum N(\mathfrak{a})^{-s}$, where $\mathfrak{a}$ runs over all integral ideals of F belonging to the ray class c. Then the Stark-Shintani ray class invariant $X_\mathfrak{f}(c)$ is defined by

$$X_\mathfrak{f}(c) = \exp(\zeta'_F(0, c) - \zeta'_F(0, c\nu)) \tag{1}$$

(Stark [12], [13], Shintani [11]). Put $X_\mathfrak{f}(c, G) = \prod_{g\in G} X_\mathfrak{f}(cg)$.

CONJECTURE ([12], [13], [11]). For some positive rational integer m, $X_\mathfrak{f}(c, G)^m$ is a unit of M ($\forall c \in H_F(\mathfrak{f})/G$). Moreover, $\{X_\mathfrak{f}(c, G)^m\}^{\sigma(c_0)} = X_\mathfrak{f}(cc_0, G)^m$ ($\forall c, c_0 \in H_F(\mathfrak{f})/G$), where σ is the Artin isomorphism of $H_F(\mathfrak{f})/G$ onto the Galois group $\mathrm{Gal}(M/F)$.

Denote by M^+ the maximal totally real subfield of M. Then Shintani proved that the conjecture is true if M^+ is abelian over the rational number field $\boldsymbol{Q}$ ([11]). In our previous paper, we have studied the integer m in the conjecture when M^+ is abelian over $\boldsymbol{Q}$, and we have constructed abelian extensions M of F with the following property (P) for an odd prime number p (cf. Theorem 1, Propositions 8, 9, 10 and 13 of [9]):