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SURFACES OF REVOLUTION WITH CONSTANT MEAN CURVATURE IN LORENTZ-MINKOWSKI SPACE

JUN-ICH HANO AND KATSUMI NOMIZU

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In an old paper [1] Delaunay proved that the profile curve of a surface of revolution with nonzero constant mean curvature in Euclidean 3-space can be described as the locus of a focus when a quadratic curve is rolled along the axis of revolution. This result was restudied by Kenmotsu [4] and Hsiang and Yu [3] for generalizations in various directions. In the present paper we shall examine the problem in Lorentz-Minkowski 3-space by studying spacelike surfaces of revolution.

In Lorentz-Minkowski 3-space the axis of revolution is either spacelike or timelike or null. In the first two cases, we can prove the results of the same kind as Delaunay's except that the nature of quadrics needs special attention. In the third case, we can determine the profile curves completely without giving a geometric interpretation. In any case, we are interested in the surfaces up to congruence by a Lorentz transformation.

In what follows, $\{x, y, z\}$ is a Lorentz coordinate system for which the metric of the space is $dx^2 + dy^2 - dz^2$. Our starting point is the lemma in [2], Section 1.

1. Surfaces of revolution with spacelike axis. Here we deal with a surface of the form

$$(x(s),\ z(s)sh\,t,\ z(s)ch\,t)\ ,$$

where s is an arc-length parameter of the profile curve $(x(s), z(s))$ in the xz-plane and $z < 0$. The principal curvatures of the surface are given by $\ddot{x}/\dot{z}$ and $\dot{x}/z$, where $\dot{x} = dx/ds$, $\ddot{x} = d^2x/ds^2$, etc.

Adopting the method in [3] we now discuss the rolling of a curve in the xz-plane. Let Γ be a smooth curve in the xz-plane given by a timelike vector-valued function

$$x = r(\theta)sh\,\theta\ ,\quad z = r(\theta)ch\,\theta \tag{1}$$

We assume that $r > 0$ and that the tangent vector of Γ is always spacelike, that is, $r^2 - r'^2 > 0$, where the prime denotes $d/d\theta$. Let Ω