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A REMARK ON THE RICCI CURVATURE OF ALGEBRAIC SURFACES OF GENERAL TYPE

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Introduction and Preliminaries. In Aubin [1], it was shown that if M is a compact complex manifold with negative first Chern class, then there is a unique Einstein-Kaehler metric on M. On the other hand, manifolds with negative first Chern class belong to the class of algebraic manifolds of general type. By the Kodaira embedding theorem, the negativity of the first Chern class is equivalent to the ampleness of the canonical bundle K, i.e., $|mK|$ gives a projective embedding for m large. Now let M be a projective manifold of dimension n. M is said to be of general type if the plulicanonical bundles have sufficiently many sections in the sense that the dimension of the image under the rational map given by the linear system $|mK|$ for m large is equal to the dimension of M, or equivalently

$$\limsup_{n\to\infty} \dim H^0(M, \mathcal{O}(mK))/m^n > 0 .$$

The aim of this note is to extend Aubin's theorem to the case of general type in dimension two. As an application, we give a differential geometric proof of the Miyaoka inequality: $3c_2 \geqq c_1^2$, for surfaces of general type. Our proof implies that if M is a surface of general type whose canonical bundle is not ample, then the strict inequality $3c_2 > c_1^2$ holds. Hence an algebraic surface of general type M is covered by the ball in $\boldsymbol{C}^2$ holomorphically if and only if the equality $3c_2(M) = c_1(M)^2$ holds. Miyaoka also proved this result in [9] by showing that there are no rational curves in M if $3c_2(M) = c_1(M)^2$ holds, using algebro-geometric methods.

In our proof, the following observation due to Kodaira is essential: "If M is a minimal surface of general type, then the canonical bundle K is ample if and only if there are no (-2)-curves", where a (-2)-curve means a non-singular rational curve with self-intersection number -2. Now let $\mathscr{E}$ be the union of all (-2)-curves in M. $\mathscr{E}$ is characterized as the set of all irreducible curves in M which do not meet the canonical divisor. Hence $\mathscr{E}$ is an obstruction to the existence of a