Tôhoku Math. Journ.
36 (1984), 377-384.

A THEOREM OF BERNSTEIN TYPE FOR MINIMAL SURFACES IN R^4

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(Received June 27, 1983)

1. Introduction. Let $u: \boldsymbol{R}^2 \to \boldsymbol{R}$ be a function the graph of which is a minimal surface in $\boldsymbol{R}^3$. Then u is a linear function by the classical theorem of Bernstein.

On the other hand, the same conclusion does not hold in the case of codimension greater than one. The graph of an entire holomorphic function on $\boldsymbol{C}$ is a minimal surface of $\boldsymbol{R}^4 = \boldsymbol{C}^2$ which is not necessarily a plane.

Recently a generalization of Bernstein's theorem was proved.

THEOREM (do Carmo and Peng [1], Fischer-Colbrie and Schoen [3]). *Complete orientable stable minimal surfaces in $\boldsymbol{R}^3$ are planes.*

A minimal surface is called *stable* if the second variation is non-negative for every normal vector field on M with compact support. This theorem suggests that the essential property is the stability. In fact, the minimal graphs of codimension one are stable (Federer [2]), whereas those of codimension greater than one are not necessarily stable (Lawson and Osserman [5], Kawai [4]).

Since the graph of a holomorphic function on $\boldsymbol{C}$ is stable, it is quite natural to ask whether or not the complete orientable stable minimal surfaces in $\boldsymbol{R}^4$ are congruent to the complex submanifolds of $\boldsymbol{C}^2 = \boldsymbol{R}^4$, i.e., transformed to the complex submanifolds of $\boldsymbol{C}^2 = \boldsymbol{R}^4$ by the isometries of $\boldsymbol{R}^4$.

The purpose of this paper is to give a partial answer to this question, i.e., to prove the following theorem.

THEOREM. *Let M be a minimal surface in $\boldsymbol{R}^4$ which is a graph of a function defined on the whole plane $\boldsymbol{R}^2$. Suppose that M is stable. Then M is a plane or the graph of a holomorphic function or the graph of an antiholomorphic function with respect to a fixed identification $\boldsymbol{R}^2 = \boldsymbol{C}$. Hence M is congruent to a complex submanifold of $\boldsymbol{C}^2 = \boldsymbol{R}^4$.*

To prove this theorem, we shall show that the second variation is negative for some normal vector field on M with compact support if M