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A REMARK ON MINIMAL FOLIATIONS OF CODIMENSION TWO

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0. Introduction. A foliation $\mathscr{F}$ of a closed Riemannian manifold W is minimal if the leaves are minimal submanifolds of W. A foliation is taut if there is a metric on W for which the foliation is minimal.

Sullivan [S], Rummler [R] and Haefliger [H] found geometrical and topological characterizations of these foliations. A codimension one oriented foliation is taut if and only if every compact leaf is cut out by a closed transversal (Sullivan). For general codimension there is a necessary and sufficient condition for $\mathscr{F}$ to be taut that depends only on the holonomy pseudo group of the foliation (Haefliger). If the leaves of $\mathscr{F}$ are all compact then $\mathscr{F}$ is taut if and only if $\mathscr{F}$ is stable (Rummler).

Recently, Oshikiri [O], proved that for $\mathscr{F}$ of codimension one and W with non-negative Ricci curvature tensor, $\mathscr{F}$ minimal implies that $\mathscr{F}$ and $\mathscr{F}^{\perp}$ are totally geodesic, where $\mathscr{F}^{\perp}$ denotes the normal flow to $\mathscr{F}$. In particular, $\mathscr{F}$ is defined by a closed form.

In this paper we generalize this theorem for the case of codimension two. Precisely, we prove the following:

THEOREM. *Let W^{n+2} be an oriented closed $(n+2)$-dimensional Riemannian manifold and $\mathscr{F}_1$ a minimal, codimension two C^∞ foliation of W. Suppose the normal distribution of $\mathscr{F}_1$, say $\mathscr{F}_2$, is C^∞ and integrable and that both $\mathscr{F}_1$ and $\mathscr{F}_2$ are orientable.*

(1) *If* $\mathrm{Ricc}(W) > 0$ *then* $\varepsilon(\mathscr{F}_2) \neq 0$.

(2) If $\mathrm{Ricc}(W) \geqq 0$ *then either* $\mathscr{F}_1$ *is totally geodesic or* $\varepsilon(\mathscr{F}_2) \neq 0$. (*Both can occur simultaneously.*)

(3) *If W has non-negative sectional curvature then either $\varepsilon(\mathscr{F}_2) \neq 0$ or $\mathscr{F}_1$ and $\mathscr{F}_2$ are totally geodesic.* (*Both can occur simultaneously.*) *Here $\varepsilon(\mathscr{F}_2)$ denotes the Euler class of $\mathscr{F}_2$ and* $\mathrm{Ricc}(W)$ *is the Ricci curvature tensor of W.*

REMARKS.

(a) For the case of non-negative sectional curvature the theorem

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