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The Fejér-Riesz inequality for Siegel domains

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Introduction. The classical Fejér-Riesz inequality ([2]) was extended from the unit disc of the complex plane $\boldsymbol{C}$ to balls and polydiscs of $\boldsymbol{C}^n$ ([4], [9], and [10]). For unbounded domains, Hille and Tamarkin derived an analogous inequality. Let $f \in H^p(\boldsymbol{R}^2_+)$, $1 \leqq p < \infty$, where $\boldsymbol{R}^2_+$ denotes the upper half-plane $\{z \in \boldsymbol{C} \mid \operatorname{Im} z > 0\}$. Then the following holds for every $x \in \boldsymbol{R}$ ([5, Theorem 4.1]):

$$\int_{\boldsymbol{R}_+} |f(x+iy)|^p dy \leqq 2^{-1} \sup_{y>0} \int_{\boldsymbol{R}} |f(x+iy)|^p dx \,, \tag{1}$$

where $\boldsymbol{R}_+$ denotes the positive numbers. Kawata [6] and Krylov [8] showed that the main results of the Hille-Tamarkin's H^p theory are valid for all $p > 0$. The inequality (1) is also seen to hold in this case. Our purpose is to deal with this inequality in a setting of higher dimensions and a wider class of functions. We shall obtain an inequality of the same sort for functions u such that $u \geqq 0$ and $\log u$ are plurisubharmonic on certain Siegel domains in $\boldsymbol{C}^n \times \boldsymbol{C}^m$. The principal result is Theorem 1 in Section 2. Section 3 is concerned with Hardy space results.

1. Preliminaries. Let u be a real-valued function on $\boldsymbol{R}^2_+$. If $u \geqq 0$ and $\log u$ is subharmonic we shall call u a log. subharmonic function. Such functions are called functions of class PL and then basic properties are found in [11]. We shall denote by $LH^p(\boldsymbol{R}^2_+)$, $0 < p < \infty$, the class of log. subharmonic functions u satisfying the condition

$$M(u, p; \boldsymbol{R}^2_+) := \sup_{y>0} \int_{\boldsymbol{R}} u(x+iy)^p dx < \infty \,. \tag{2}$$

Let Ω be an open cone in $\boldsymbol{R}^n$ which is the interior of the convex hull of n linearly independent half-lines starting from the origin. We shall call Ω an n-polygonal cone. The tube domain with base Ω is defined by $T_\Omega = \{X + iY \in \boldsymbol{C}^n \mid X \in \boldsymbol{R}^n,\ Y \in \Omega\}$. Let u be a real-valued function defined on T_Ω and $u \geqq 0$. If $\log u$ is plurisubharmonic we shall call u a log.

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