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DIMENSION OF SPACES OF VECTOR VALUED AUTOMORPHIC FORMS ON THE UNITARY GROUP $SU(2, 1)$

HISASHI KOJIMA

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Introduction. The purpose of this paper is to investigate the dimension of the spaces of the vector valued holomorphic automorphic forms defined on the domain $D=\{(z, w)\in \boldsymbol{C}^2 \mid \delta(\bar{z}-z)-|w|^2>0\}$, where δ is an element of an imaginary quadratic field F with $\bar{\delta}=-\delta(\neq 0)$. Let $\Gamma(N)$ be an arithmetic subgroup of $G_{\boldsymbol{R}}$ defined in §1. Let ρ be an irreducible polynomial representation of $GL_2(\boldsymbol{C})$ of degree $m+1$. Consider a $\boldsymbol{C}^{m+1}$-valued holomorphic function $f(Z)$ on D satisfying

$$f(\gamma(Z))=\rho(J(\gamma, Z))f(Z)$$

for every $Z\in D$ and for every $\gamma\in\Gamma(N)$, where $J(\gamma, Z)$ is the canonical automorphy factor on $\Gamma(N)\times D$. Denote by $S_\rho(\Gamma(N))$ the space of all such forms. In [3], Cohn calculated the dimension of $S_\rho(\Gamma')$ in the case where $F=\boldsymbol{Q}(\sqrt{-1})$, $\delta=\sqrt{-1}$, $\rho(g)=\det(g)^k$ and $\Gamma'=G_{\boldsymbol{Q}}\cap M_3(\mathfrak{O}_F)$ (see §1 for $G_{\boldsymbol{Q}}$). In this paper we try to extend his results to the case where F is an imaginary quadratic field of class number one, ρ is an arbitrary irreducible representation and $\Gamma(N)$ is a principal congruence subgroup of $\Gamma(1)$.

§1 is devoted to classifying the elements of $\Gamma(N)$, using several methods of Cohn. In §2, we construct a good fundamental domain for $\Gamma(1)$. In §3, applying the method of Selberg [8] and Godement [4], we reduce the computation of $\dim S_\rho(\Gamma(N))$ to that of certain integrals. In the last section, using a method similar to those of Shimizu [9] and Morita [7], we establish the following theorem:

THEOREM. *Suppose that F is an imaginary quadratic field of class number one and $k\geqq m+6$. Then*

$$\begin{aligned}\dim S_\rho(\Gamma(N)) &= \left\{2^{k+m-1}\pi^2(-i\delta)(2k+2m-3)!!((2k+2m-2)!)^{-1}\sum_{l=0}^{m}{}_mC_l(m-l)!(l+k-3)!\right\}^{-1}\\ &\quad |\Gamma/\Gamma(N)|\left\{(m+1)\operatorname{vol}(\Gamma\backslash D)+\delta^2 n_0(|\delta|^2 n_1^2)^{-1}\zeta(2)\operatorname{vol}(\boldsymbol{C}/\delta\mathfrak{m})|E(F)|^{-1}\right.\\ &\quad \left.\times\sum_{j=0}^{m}((k+j-1)(k+j-2))^{-1}\right\}.\end{aligned}$$