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ON ZETA-FUNCTIONS AND CYCLOTOMIC Z_p-EXTENSIONS OF ALGEBRAIC NUMBER FIELDS

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1. In Tate [5] and Turner [7], the following result is proved:

THEOREM. *Let k, k' be function fields in one variable over a finite constant field $\boldsymbol{F}$ and ζ_k, $\zeta_{k'}$ Dedekind zeta-functions of k, k'. Let C, C' be complete non-singular curves defined over $\boldsymbol{F}$ with function fields isomorphic to k, k' and $J(C)$, $J(C')$ the Jacobian varieties of C, C'. Then the following are equivalent*:

(1) $\zeta_k = \zeta_{k'}$.

(2) *$J(C)$ and $J(C')$ are $\boldsymbol{F}$-isogenous.*

In the present paper, we shall investigate the situation which arises when we replace the function fields by the algebraic number fields. In [2] and [3], Iwasawa discussed analogues of Jacobian varieties in this situation. We shall see that these analogues play some roles in this question.

Let $\boldsymbol{Q}$ be the rational number field, k, k' finite algebraic extensions of $\boldsymbol{Q}$ and ζ_k, $\zeta_{k'}$ the Dedekind zeta-functions of k and k', respectively. Perlis [4] gave interesting consequences from $\zeta_k = \zeta_{k'}$. Using his method, we shall obtain the following results:

Let p be a prime number, $k(p)$ the maximal abelian pro-p-extension of k and $G_k(p)$ the Galois group of $k(p)$ over k. For these and also for other notations which will be introduced afterwards, we adopt similar notations for k'. Let $\boldsymbol{Z}_p$ be the p-adic integer ring and k_∞ the cyclotomic $\boldsymbol{Z}_p$-extension of k. We shall prove that $\zeta_k = \zeta_{k'}$ implies $G_k(p) \cong G_{k'}(p)$ and $G_{k_\infty}(p) \cong G_{k'_\infty}(p)$ for almost all prime numbers p. Let $\tilde{k}_\infty$ the maximal unramified abelian pro-p-extension of k_∞ and $Y_k(p)$ the Galois group of $\tilde{k}_\infty/k_\infty$. Let A and A' be the p-primary subgroups of ideal class groups of k_∞ and k'_∞, respectively. Let $X_k(p)$ be the Pontrjagin dual of the discrete group A. Let α_p be a primitive p-th root of 1. We shall prove that $\zeta_k = \zeta_{k'}$ implies $X_{k(\alpha_p)}(p) \cong X_{k'(\alpha_p)}(p)$ and $Y_{k(\alpha_p)}(p) \cong Y_{k'(\alpha_p)}(p)$ for almost all prime numbers p. The duals of $X_{k(\alpha_p)}(p)$ and $Y_{k(\alpha_p)}(p)$ are regarded as analogies of the Jacobian variety in our situation (cf. [2], [3]), so that this can be interpreted as an analogue of the fact that (1) implies (2) in the