Tôhoku Math. Journ.
36 (1984), 541–554.

MINIMAL FOLIATIONS ON LIE GROUPS

RYOICHI TAKAGI AND SHINSUKE YOROZU

(Received September 6, 1983)

1. Introduction. The study of minimal foliations, totally geodesic foliations and foliations with bundle-like metrics may be of interest to us in the meaning that they are geometric properties combined foliated structures with Riemannian structures. The foliated Riemannian manifold with a bundle-like metric was defined by Reinhart [15] and is discussed by him and others ([7], [10], [11]). The Riemannian submersion ([2], [13]) is a special case of this conception. The foliated Riemannian manifold with totally geodesic leaves is discussed by Dombrowski [1], Ferus [3], Johnson and Whitt [8], Tanno [18] and others. This case often appears in the differential geometry. Recently, Haefliger [5], Kamber and Tondeur [9], Rummler [16], Sullivan [17] and many people discuss the foliated Riemannian manifold with minimal leaves.

In this paper we define a foliation on a Lie group. For a Lie group G, we take a Lie subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$ associated to G and a left invariant Riemannian metric $\langle\ ,\ \rangle$. Then we have a foliated Riemannian manifold $(G, \langle\ ,\ \rangle, \mathscr{F}(\mathfrak{h}))$. On $(G, \langle\ ,\ \rangle, \mathscr{F}(\mathfrak{h}))$, we discuss the totally geodesicness and minimality of leaves and bundle-like-ness of the metric. We have many interesting examples, for instance, foliated Riemannian manifolds with minimal, not totally geodesic leaves. From these examples, we may remark that it is not able to extend Oshikiri's theorem [14] to the case of codimension $\geqq 2$.

2. Preliminaries. Let (M, g) be an n-dimensional Riemannian manifold M with a Riemannian metric g. The objects under consideration are of class C^∞. Let $\{e_A\}$ be a local orthonormal frame field in M and $\{w^A\}$ be the dual coframe field. Here and hereafter, indices $A, B, \cdots$ run from 1 to n. The connection forms w_B^A on M associated with $\{e_A\}$ are uniquely defined by

$$dw^A = -\sum_B w_B^A \wedge w^B\ , \quad w_B^A + w_A^B = 0\ .$$

We set $dw^A = -\sum_{B,C} \Gamma_{BC}^A w^B \wedge w^C$, $\Gamma_{BC}^A + \Gamma_{CB}^A = 0$, then w_B^A are given by

$$w_B^A = \sum_C (-\Gamma_{BC}^A + \Gamma_{AB}^C - \Gamma_{CA}^A) w^C\ . \tag{2.1}$$