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LIMITS OF SEQUENCES OF RIEMANN SURFACES REPRESENTED BY SCHOTTKY GROUPS

(To Professor Yukio Kusunoki on the occasion of his 60th birthday)

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0. Introduction. In this paper, we state an application of the interchange operators introduced in the previous paper [8]. We consider the following problem. We give a point τ in an augmented Schottky space $\hat{\mathfrak{S}}_g^*(\tilde{\Sigma}_0)$ associated with $\tilde{\Sigma}_0$, which represents a compact Riemann surface S with nodes. Then for any sequence of points $\{\tau_n\}$ in the Schottky space $\mathfrak{S}_g(\tilde{\Sigma}_0)$ associated with $\tilde{\Sigma}_0$ tending to the point τ, does the Riemann surfaces $S(\tau_n)$ represented by τ_n converge to S as marked surfaces as $n \to \infty$?

The answer to this problem is negative in the general case, namely in the case where $\tilde{\Sigma}_0$ is a basic system of Jordan curves (see § 1.2 for the definition). However the answer is affirmative in a special case, namely in the case where $\tilde{\Sigma}_0$ is a standard system of Jordan curves (see § 1.2 for the definition). Now the following question arises: To what Riemann surfaces does the sequence of Riemann surfaces $\{S(\tau_n)\}$ converge as marked surfaces as $n \to \infty$ in the general case? The answer is the main result (Theorem 2 in § 6) in this paper.

We use the same notation and terminologies as in [8]. In § 1, we will define convergence of Riemann surfaces, and in § 2, we will show the following: For any point τ in an augmented Schottky space, there exists a sequence of points $\{\tau_n\}$ in the Schottky space tending to τ such that the sequence of Riemann surfaces $\{S(\tau_n)\}$ represented by τ_n converges to the Riemann surface $S(\tau)$ represented by τ as marked surfaces as $n \to \infty$. In § 3, we will construct a new surface from a given surface. From § 4 through § 6, we will state and prove the main theorem. In § 7, we will explain the result by an example.

1. Definitions and terminologies

1.1. We use the same notation and terminologies as in the previous papers [7, 8].

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