Tôhoku Math. Journ.
36 (1984), 485-503.

ANALYSIS OF DYADIC STATIONARY PROCESSES USING THE GENERALIZED WALSH FUNCTIONS

YASUSHI ENDOW

(Received March 28, 1983)

Abstract. This paper deals with continuous-parameter dyadic stationary processes. A necessary and sufficient condition for such a process to assume its spectral representation in terms of the generalized Walsh functions is given. The representation plays an important role in the analysis of such a process: we discuss laws of large numbers, sampling theorem, and the relationship between the dyadic stationary processes with spectral densities and linear dyadic processes.

The existence of a spectral representation shows the possibility of an analysis of dyadic stationary processes similar to that of ordinary stationary processes.

1. Introduction. In communication theory and systems engineering, analysis of signals contaminated by random noise is very important. The general problem is difficult; so it is usually preferable to restrict attention to certain classes of signals.

Recently much attention has been paid to a class of signals that are called dyadic stationary processes [7], [17], [19], [20], [22], [23]. Most discussions, however, have confined themselves to the discrete-parameter case. This is perhaps due to the difficulty arising from the discontinuity of the Walsh functions, in terms of which the processes are represented.

Analysis of the dyadic stationary processes is most readily carried out using the Walsh functions rather than the exponential functions commonly used in the ordinary stationary processes. Many researchers, among others Walsh [28], Fine [8]-[10], Chrestenson [6], Mogenthaler [18], Paley [21], Selfridge [25] and Yano [29], have contributed to Walsh-Fourier analysis, while dyadic calculus involving dyadic derivatives was developed by Gibbs [11], [12], Butzer and Wagner [1]-[3], and Wagner [27].

In this paper we consider continuous-parameter dyadic stationary processes. For their spectral representations, the theory of the generalized Walsh functions is a necessity. In 2 we shall briefly state the definitions and some properties of the generalized Walsh functions which are used frequently afterwards. In 3 we introduce a continuous-parameter dyadic stationary process and give the spectral representations of its covariance