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AXIOMS FOR STIEFEL-WHITNEY HOMOLOGY CLASSES OF Z_2-EULER SPACES

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1. Introduction and the statement of results. In [2], Blanton and Schweitzer gave an axiomatic characterization for Stiefel-Whitney classes or Stiefel-Whitney homology classes of smooth manifolds, and raised a question of axiomatic characterizations of these classes for other categories, for example, categories of PL-manifolds, topological manifolds or Euler spaces. In this paper we give an answer to this question for $\boldsymbol{Z}_2$-Euler spaces (cf. [5], [8]).

Let X and Y be $\boldsymbol{Z}_2$-Euler spaces and let $\varphi: Y \to X$ be a PL-embedding. We call φ a regular embedding if $\dim X = \dim Y$, $\varphi(Y)$ is closed in X, $\varphi(\operatorname{Int} Y) \cap \partial X = \emptyset$ and $\varphi|\operatorname{Int} Y$ is an open map, where $\operatorname{Int} Y = Y - \partial Y$.

Let $H_*^{\inf}$ denote the homology theory of infinite chains. Given a regular embedding $\varphi: Y \to X$, we define a homomorphism $\varphi^\#: H_*^{\inf}(X, \partial X; \boldsymbol{Z}_2) \to H_*^{\inf}(Y, \partial Y; \boldsymbol{Z}_2)$ by $\varphi^\# = (\varphi_*)^{-1} \circ i_*$, where $i_*: H_*^{\inf}(X, \partial X; \boldsymbol{Z}_2) \to H_*^{\inf}(X, X - \varphi(\operatorname{Int} Y); \boldsymbol{Z}_2)$ is the homomorphism induced from the identity $i: (X, \partial X) \to (X, X - \varphi(\operatorname{Int} Y))$. Note that $\varphi_*: H_*^{\inf}(Y, \partial Y; \boldsymbol{Z}_2) \to H_*^{\inf}(X, X - \varphi(\operatorname{Int}); \boldsymbol{Z}_2)$ is an isomorphism by the excision property. Therefore $\varphi^\#$ is well defined.

Let $\mathscr{E}$ be the category whose objects are $\boldsymbol{Z}_2$-Euler spaces and whose morphisms are regular embeddings. Let $\mathscr{S}$ be a full subcategory of $\mathscr{E}$. Consider a homology class

$$S_*(X) = S_0(X) + S_1(X) + \cdots + S_n(X) \quad \text{in} \quad H_*^{\inf}(X, \partial X; \boldsymbol{Z}_2),$$

where n is the dimension of X, satisfying the following axioms:

AI. For every object X of $\mathscr{S}$ and every integer $i \geqq 0$, there is a homology class $S_i(X)$ in $H_i^{\inf}(X, \partial X; \boldsymbol{Z}_2)$.

AII. If $\varphi: Y \to X$ is a morphism of $\mathscr{S}$, then $S_*(Y) = \varphi^\# S_*(X)$.

AIII. $S_*(X \times Y) = S_*(X) \times S_*(Y)$ for every objects X, Y of $\mathscr{S}$, such that $X \times Y$ is an object of $\mathscr{S}$.

AIV. For every integer $n \geqq 0$, $S_*(\boldsymbol{P}^n) = s_*(\boldsymbol{P}^n)$, where $s_*(\boldsymbol{P}^n)$ is the Stiefel-Whitney homology class of the n-dimensional real projective space $\boldsymbol{P}^n$.

We call $S_*(X)$ an axiomatic Stiefel-Whitney homology class of X in