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SQUARE-INTEGRABLE HOLOMORPHIC FUNCTIONS ON A CIRCULAR DOMAIN IN C^n

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0. Introduction. In the preceding paper [2], square-integrable holomorphic n-forms on an n-dimensional complex manifold are studied, and invariants $\mu_{0,m}$ are introduced. The purpose of this paper is to examine how $\mu_{0,m}$ are expressed when the manifold is a circular domain in the n-dimensional complex Euclidean space C^n, and to provide several examples concerning these invariants.

Let D be a circular domain in C^n which is not necessarily bounded. Let $H(D)$ be the Hilbert space of all square-integrable holomorphic functions on D, and for every integer m, let $H_m(D)$ be the subspace of $H(D)$ whose elements are m-homogeneous on D (see Definition 1.1). Then $H_m(D)$ are mutually orthogonal. If D is proper, then $H_m(D) = \{0\}$ for $m < 0$, and all elements of $H_m(D)$ for $m \geqq 0$ are actually homogeneous polynomials of degree m. Now, suppose that D is proper and has a finite volume $V(D)$. Let $K(z, \bar{w}) = \sum_{m=0}^{\infty} K_m(z, \bar{w})$ be the Bergman kernel of D, where K_m are homogeneous polynomials of degree m with respect to each of the variables z and $\bar{w}$. Then it is shown that

$$\mu_{0,m}((\partial_v)_O) = V(D)(m!)^2 K_m(v, \bar{v})$$

for $v \in C^n$, where $\partial_{(v^1,\cdots,v^n)} = \sum_j v^j \partial/\partial z^j$ (Theorem 2.2). Furthermore, if D is bounded, then every polynomial K_m is written as follows (Corollary 2.4):

$$K_m(z, \bar{w}) = (z^{I_1}, \cdots, z^{I_N})\bar{G}^{-1}(w^{I_1}, \cdots, w^{I_N})^* ,$$

where $(I_1, \cdots, I_N)$ $\left(N = \binom{n+m-1}{m}\right)$ is a numbering of the indices of the set $\{(i_1, \cdots, i_n) \in Z_+^n;\ i_1 + \cdots + i_n = m\}$ and $G = ((z^{I_i}, z^{I_j}))_{i,j}$ is the Gram matrix of the system $(z^{I_1}, \cdots, z^{I_N})$ of monomials with respect to the inner product on $H(D)$.

It is well-known ([7], [10]) that when a domain carries a Bergman metric g, the holomorphic sectional curvature of g does not exceed 2. In § 3, we see the following from examples:

(i) There exists a domain D in C^2 with positive, finite dimensional $H(D)$. Moreover, there exists a domain in C^2 for which the holomorphic