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EXPOSED POINTS AND EXTREMAL PROBLEMS IN H^1, II

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ABSTRACT. If $\phi \in L^\infty$, we denote by T_ϕ the functional defined on the Hardy space H^1 by

$$T_\phi(f)=\int_{-\pi}^{\pi} f(e^{i\theta})\phi(e^{i\theta})d\theta/2\pi .$$

Let S_ϕ be the set of functions in H^1 which satisfy $T_\phi(f)=\|T_\phi\|$ and $\|f\|_1 \leqq 1$. If S_ϕ is not empty and weak*-compact, a description of S_ϕ was given in the first part of this paper. In this paper, the structure of S_ϕ is studied generally. Moreover, we give a characterization of exposed points, that is, g in H^1 such that $S_\phi=\{g\}$ for some ϕ.

1. Introduction. Let U be the open unit disc in the complex plane and let ∂U be the boundary of U. If f is analytic in U and $\int_{-\pi}^{\pi} \log^+|f(re^{i\theta})|d\theta$ is bounded for $0 \leqq r < 1$, then $f(e^{i\theta})$, which we define to be $\lim_{r\to 1} f(re^{i\theta})$, exists almost everywhere on ∂U. If

$$\lim_{r\to 1}\int_{-\pi}^{\pi} \log^+|f(re^{i\theta})|d\theta = \int_{-\pi}^{\pi} \log^+|f(e^{i\theta})|d\theta ,$$

then f is said to be in the class N_+. The set of all boundary functions in N_+ is denoted by N_+ again. For $0 < p \leqq \infty$, the Hardy space H^p is defined as $N_+ \cap L^p$. If $1 \leqq p \leqq \infty$, it coincides with the space of functions in L^p whose Fourier coefficients with negative indices vanish. If h in N_+ has the form

$$h(z) = \exp\left\{\int_{-\pi}^{\pi} \frac{e^{it}+z}{e^{it}-z}\log|h(e^{it})|dt/2\pi + i\alpha\right\} \qquad (z \in U)$$

for some real α, then h is called an outer function. We call q in N_+ an inner function if $|q(e^{i\theta})| = 1$ a.e. on ∂U. Each nonzero f in H^1 has a unique factorization of the form $f = qh$, where q is an inner function and h is an outer function.

If $\phi \in L^\infty$, we denote by T_ϕ the functional defined on H^1 by

$$T_\phi(f) = \int_{-\pi}^{\pi} f(e^{i\theta})\phi(e^{i\theta})d\theta/2\pi .$$

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