Tôhoku Math. Journ.
37 (1985), 251-263.

NONLINEAR SEMIGROUP FOR THE UNNORMALIZED CONDITIONAL DENSITY

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(Received March 7, 1984)

1. Introduction. We are concerned with partially observable control problems. Let X_t be a state process being controlled, Y_t an observation process and U_t an admissible control defined on a probability space (Ω, F, P). The process X_t and Y_t are governed by the following stochastic differential equations:

$$dX_t = b(X_t, U_t)dt + \sigma(X_t)dW_t \quad 0 < t \leqq T\,, \tag{1.1}$$

$$dY_t = h(X_t)dt + d\tilde{W}_t \quad 0 < t \leqq T\,, \tag{1.2}$$

where W_t and $\tilde{W}_t$ are independent Wiener processes with values in $\boldsymbol{R}^N$ and $\boldsymbol{R}^M$, respectively (for simplicity, we let $M = 1$ here).

Our object is to minimize

$$J = Ef(X_T) \tag{1.3}$$

by a suitable choice of an admissible control, where f is a given cost function. Define Z_t by

$$Z_t = \exp\left[\int_0^t h(X_s)dY_s - (1/2)\int_0^t |h(X_s)|^2 ds\right].$$

Then, by Girsanov's formula, Y_t and W_t turn out as independent Wiener processes under the new probability measure $\hat{P}$ defined by $d\hat{P} = Z_T^{-1}dP$. In partially observable control problems, an admissible control U_t is usually measurable with respect to $\sigma_t(Y)$ (the σ-field generated by the observation process Y_s for $0 \leqq s \leqq t$). But, in this note we apply the same idea of admissibility as that in Fleming and Pardoux [5], namely we merely require that U_t is independent of W and $Y_r - Y_t$ for $r \geqq t$. Let F_t denote $\sigma_t(Y, U)$ and $L(u)$ be the infinitesimal generator of X_t with a constant control u. Bensoussan [1] and Pardoux [9] showed that the unnormalized conditional probability $P(t, \omega)$, defined by

$$\hat{E}[g(X_t)Z_t|F_t](\omega) = \int_{\boldsymbol{R}^N} g(x)P(t, \omega)(dx)$$

for any bounded Borel function g on $\boldsymbol{R}^N$, has a density $p(t, x, \omega)$ under mild assumptions on b, σ and h. Furthermore, $p(t)$ is regarded as a