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A CONDITION FOR ISOPARAMETRIC HYPERSURFACES OF S^n TO BE HOMOGENEOUS

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1. Introduction. Let M be a connected hypersurface of the n-dimensional sphere S^n of radius 1. $O(n+1)$ acts on S^n as an isometry group. M is said to be homogeneous if it is an orbit of a certain subgroup of $O(n+1)$. M is said to be isoparametric if it has constant principal curvatures. If M is homogeneous then it is isoparametric. E. Cartan investigated the converse problem and he gave an affirmative answer in some special cases ([2], [3], [4], [5]). But, recently, Ozeki and Takeuchi gave examples of isoparametric hypersurfaces which are not homogeneous in [8], using a result of Münzner [7]. On the other hand, homogeneous hypersurfaces of S^n are investigated in detail by Hsiang and Lawson [6] and by Takagi and Takahashi [10].

In the present paper, we give an additional differential geometric condition for isoparametric hypersurfaces of S^n to be homogeneous, using the result to Münzner. Our main results are the following Theorems A and B. To state them, we need some notations. Let $T_1, \cdots, T_r$ and T be tensor fields on a manifold. T is said to be generated by $T_1, \cdots, T_r$ if T is a constant linear combination of tensor fields, each of which is a tensor product of some members of $T_1, \cdots, T_r$ or its contraction. We denote this fact by $T = P(T_1, \cdots, T_r)$. Let M be a Riemannian manifold. Let M_p and M_q be the tangent spaces at $p, q \in M$. Then M_p and M_q are vector spaces with the inner products given by the Riemannian metric. A linear isometry L of M_p onto M_q is extended naturally to an isomorphism of the tensor algebra $T(M_p)$ onto $T(M_q)$, which is denoted also by L. For an oriented hypersurface M of S^n, we denote by G, H, ∇ and $\nabla^m H$ the first and second fundamental forms, the covariant differentiation and the m-th covariant differential, respectively. By G^{-1}, we denote the inner product for 1-forms on M induced naturally from G.

THEOREM A. *Let M be an oriented isoparametric hypersurface of S^n with g distinct principal curvatures. Then, for any $m \geqq g-1$, $\nabla^m H$*

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