Tôhoku Math. Journ.
37 (1985), 225-240.

CORONA THEOREM WITH BOUNDS FOR FINITELY SHEETED DISKS

(Dedicated to Professor Yukio Kusunoki on the occasion of his 60th birthday)

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(Received February 3, 1984)

The primary purpose of this paper is to show that the corona theorem with bounds is valid for finitely sheeted disks possibly with infinitely many branch points where the bounds are dominated by universal constants depending only on their sheet numbers. As a consequence the corona theorem with bounds is valid for finite Riemann surfaces where the bounds are dominated by universal constants depending only on their Euler characteristics.

We start by fixing terminology before stating our main result precisely. An n-tuple $\{f_j\}$ of functions $f_1, \cdots, f_n$ in $H^\infty(R)$ of the family of bounded holomorphic functions on a Riemann surface R is referred to as a *corona datum* of length n in $\boldsymbol{N}$, the set of positive integers, and of lower bound δ in the interval $(0, 1)$, or simply of *index* (n, δ), if the following two conditions are satisfied: $\max_{1\leqq j\leqq n}(\sup_R |f_j|) \leqq 1$ and $\inf_R(\sum_{j=1}^n |f_j|) \geqq \delta$. An n-tuple $\{g_j\}$ of functions $g_1, \cdots, g_n$ in $H^\infty(R)$ is said to be a *corona solution* of the datum $\{f_j\}$ if $\sum_{j=1}^n f_j g_j = 1$. The quantity $C(R; n, \delta)$ in $(0, \infty]$ given by

$$C(R; n, \delta) = \sup_{\{f_j\}}(\inf_{\{g_j\}}(\max_{1\leqq j\leqq n}(\sup_R |g_j|))) \tag{1}$$

will be referred to as the *Gamelin constant* of R of index (n, δ) in $\boldsymbol{N}\times(0, 1)$ where the first supremum is taken with respect to corona data $\{f_j\}$ of index (n, δ) on R and the next infimum is taken with respect to corona solutions $\{g_j\}$ of each fixed datum $\{f_j\}$ under the usual convention that $\inf_{\{g_j\}} = \infty$ if there exist no corona solutions $\{g_j\}$ of the datum $\{f_j\}$. Since the quantity was first systematically considered for plane regions by Gamelin [6], we attach the name to the quantity for the convenience of references. We should mention that the quantity was also considered for plane regions by Behrens [2].

We say that the *corona theorem* is valid on a Riemann surface R if

*) This work was partly supported by Grant-in-Aid for Scientific Research, No. 454027, Japanese Ministry of Education, Science and Culture.