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LARGE TIME ASYMPTOTICS FOR FUNDAMENTAL SOLUTIONS OF DIFFUSION EQUATIONS

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1. Introduction. The purpose of the present paper is to give asymptotic expansions as $t\to\infty$ of the fundamental solution of a diffusion equation in R^n.

Let

$$A=\sum_{j,k=1}^{n} a_{jk}(x)\partial_j\partial_k+\sum_{j=1}^{n} b_j(x)\partial_j+c(x) \tag{1.1}$$

be an elliptic operator satisfying the following condition (A). Here $\partial_j=\partial/\partial x_j$.

(A. I) There exists a positive constant c_0 such that $\sum_{j,k}a_{jk}(x)\xi_j\xi_k \geqq c_0|\xi|^2$ for all $x, \xi\in R^n$.

(A. II) The functions $a_{jk}(x)$, $b_j(x)$, $c(x)$ are real-valued bounded functions on R^n which are uniformly Hölder continuous with exponent θ $(0<\theta\leqq 1)$.

(A. III) There exist positive constants ρ and M such that for all $x\in R^n$

$$\sum_{j,k=1}^{n}|a_{jk}(x)-\delta_{jk}|+\sum_{j=1}^{n}\langle x\rangle|b_j(x)|+\langle x\rangle^2|c(x)|\leqq M\langle x\rangle^{-\rho}\ , \tag{1.2}$$

where δ_{jk} is Kronecker's delta and $\langle x\rangle=(1+|x|^2)^{1/2}$. Let $U(t, x, y)$ be the fundamental solution of the diffusion equation

$$\partial_t U(t, x, y)=AU(t, x, y)\ \text{in}\ (0, \infty)\times \boldsymbol{R}^n\ ,\quad U(0, x, y)=\delta(x-y)\ , \tag{1.3}$$

where $\partial_t=\partial/\partial t$ and $\delta(z)$ is the delta function. For σ in R^1, we denote by $[\sigma]$ the largest integer smaller than or equal to σ. One of our main results is the following theorem.

THEOREM 1.1. *Let $c(x)\equiv 0$ and $U(t, x, y)$ be the corresponding fundamental solution. Then for any σ with $0\leqq\sigma<\rho/2$ there hold the following formulas for all $t>1$ and $(x, y)\in R^{2n}$:*

(i) *For n odd,*

$$U(t, x, y)=\sum_{j=0}^{[\sigma]} t^{-n/2-j}U_j(x, y)+\widetilde{U}_\sigma(t, x, y)\ , \tag{1.4}$$

$$|U_j(x, y)|\leqq M_j(\langle x\rangle+\langle y\rangle)^{2j}\ , \tag{1.5}$$