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BANACH ALGEBRA RELATED TO DISK POLYNOMIALS

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Introduction. Let $\alpha \geqq 0$, and let m and n be nonnegative integers. *Disk polynomials* $R_{m,n}^{(\alpha)}(z)$ are defined by

$$R_{m,n}^{(\alpha)}(z) = \begin{cases} R_n^{(\alpha,m-n)}(2r^2-1)e^{i(m-n)\phi}r^{m-n} & \text{if} \quad m \geqq n\,, \\ R_m^{(\alpha,n-m)}(2r^2-1)e^{i(m-n)\phi}r^{n-m} & \text{if} \quad m < n\,, \end{cases}$$

where $z = re^{i\phi}$ and $R_n^{(\alpha,\beta)}(x)$ is the Jacobi polynomial of degree n and of order (α, β) normalized so that $R_n^{(\alpha,\beta)}(1) = 1$.

Denote by $A^{(\alpha)}$ the space of absolutely convergent disk polynomial series on the closed unit disk $\bar{D}$ in the complex plane, that is, the space of functions f on $\bar{D}$ such that

$$f(z) = \sum_{m,n=0}^{\infty} a_{m,n}R_{m,n}^{(\alpha)}(z) \quad \text{with} \quad \sum_{m,n=0}^{\infty} |a_{m,n}| < \infty\,,$$

and introduce a norm in $A^{(\alpha)}$ by

$$\|f\| = \sum_{m,n=0}^{\infty} |a_{m,n}|\,.$$

The space $A^{(\alpha)}$ consists of continuous functions on $\bar{D}$, since if $\sum |a_{m,n}| < \infty$ then the series $\sum a_{m,n}R_{m,n}^{(\alpha)}(z)$ converges uniformly on $\bar{D}$ by the inequality;

$$(1) \qquad |R_{m,n}^{(\alpha)}(z)| \leqq 1 \quad \text{on } \bar{D} \qquad \text{(Koornwinder [5; (5.1)])}\,.$$

Our purpose is to study some structure of the algebra $A^{(\alpha)}$.

Let $A^{(\alpha,\beta)}$ be the space of absolutely convergent Jacobi polynomial series $f(x) = \sum_{n=0}^{\infty} a_n R_n^{(\alpha,\beta)}(x)$, $\sum_{n=0}^{\infty} |a_n| < \infty$ on the closed interval $[-1, 1]$. The space $A^{(\alpha,\beta)}$ has the structure of a Banach algebra with pointwise multiplication of functions. This is proved by the nonnegativity of the linearization coefficients of products of Jacobi polynomials (see Gasper [2]) Igari and Uno [3] and Cazzaniga and Meaney [1] studied some structure of the algebra $A^{(\alpha,\beta)}$, that is, the maximal ideal space, Helson sets, spectral synthesis, etc. For the space $A^{(\alpha)}$, we will consider some of these problems. In §§1 and 2, we will show that $A^{(\alpha)}$ is a Banach algebra by the nonnegativity of the linearization coefficients of products of disk polynomials that is proved by Koornwinder [6], and then determine the maximal ideal space of $A^{(\alpha)}$. Moreover, we will show that if $\alpha \geqq 1$ and