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SPECIAL VALUES OF ZETA FUNCTIONS ASSOCIATED TO CUSP SINGULARITIES

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0. Introduction. Hirzebruch defined in [5] several geometric invariants for normal isolated singularities, in particular the ϕ-invariants for Hilbert modular cusp singularities. The invariant ϕ is the difference between the L-polynomial and the signature on the desingularization of a compact neighborhood of the singular point. He conjectured that the ϕ-invariant coincides with the invariant w, which was defined by Shimizu [11] as the special value of an L-function. This conjecture was recently proved by Atiyah, Donnelly and Singer [1]. Ehlers [4] defined and computed the ψ-invariant for Hilbert modular cusp singularities, and Satake [9], [10] generalized these to the cusp contributions for certain locally symmetric varieties, i.e., arithmetic varieties of $\boldsymbol{Q}$-rank one. From the dimension formula of Hilbert modular cusp forms, it is conjectured in [6] that the invariants ψ and ϕ coincide.

Here we consider generalized cusp singularities of Tsuchihashi [13]. Generalizing the work of Satake [8], we associate a zeta function to a pair of a nondegenerate open convex cone and a discrete group appearing in the definition of Tsuchihashi's cusp singularity. We show among other things that the special value of the zeta function gives information on the topology of the singularity, namely, the cusp contribution in odd-dimensional cases.

Let N be a free $\boldsymbol{Z}$-module of rank n (>1) and $N_{\boldsymbol{R}} := N \otimes_{\boldsymbol{Z}} \boldsymbol{R}$. Let C be a nondegenerate open convex cone in $N_{\boldsymbol{R}}$ and Γ a subgroup in the group $GL(N) := \mathrm{Aut}_{\boldsymbol{Z}}(N)$ of $\boldsymbol{Z}$-linear automorphisms of N such that C is Γ-invariant, Γ acts on $D := C/\boldsymbol{R}_{>0}$ properly discontinuously and freely, and that D/Γ is compact. Then the semi-direct product $N \cdot \Gamma$ acts on the tube domain $N_{\boldsymbol{R}} + \sqrt{-1}\, C$ in $N_{\boldsymbol{C}} := N \otimes_{\boldsymbol{Z}} \boldsymbol{C}$ properly discontinuously and freely. We get a complex manifold $(N_{\boldsymbol{R}} + \sqrt{-1}\, C)/N \cdot \Gamma$. By adding a point ∞, we can make $\{(N_{\boldsymbol{R}} + \sqrt{-1}\, C)/N \cdot \Gamma\} \cup \{\infty\}$ a complex analytic space. Tsuchihashi's cusp singularity is this point ∞. Let X be the exceptional set of a resolution of this singularity. Then $X = X_1 + \cdots + X_l$ is a toric divisor, that is, X has only normal crossings as singularities, each irreducible component X_j of X is isomorphic to an $(n-1)$-dimen-