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ON THE LITTLEWOOD-PALEY AND MARCINKIEWICZ FUNCTIONS IN HIGHER DIMENSIONS

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1. Introduction. In this paper we deal with the generalized Littlewood-Paley, Marcinkiewicz and related square functions of spherical sense in the n-dimensional space. So our functions are different from Stein's $g_\lambda^*(\boldsymbol{x}; f)$ [14. p. 99] and $\mathscr{D}_\alpha(f)(\boldsymbol{x})$ [15, p. 102].

In what follows, we shall use the following notations. $\boldsymbol{x}, \boldsymbol{\xi}, \cdots$ will denote points in the Euclidean n-space $\boldsymbol{R}^n$ $(n \geqq 2)$. In coordinate notation we write $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$; $|\boldsymbol{x}|$ denotes the length of the vector $\boldsymbol{x}$, i.e., $|\boldsymbol{x}|^2 = x_1^2 + x_2^2 + \cdots + x_n^2$; $\boldsymbol{x}' = (x_1', x_2', \cdots, x_n')$ denotes the unit vector in the direction of $\boldsymbol{x}$, i.e., $\boldsymbol{x}' = \boldsymbol{x}/|\boldsymbol{x}|$; Σ is the unit sphere, $|\boldsymbol{x}| = 1$; and $d\sigma$ is the Euclidean element of measure on Σ, hence $\int_\Sigma d\sigma = 2\pi^{n/2}/\Gamma(n/2)$.

For $f \in \mathscr{S}(\boldsymbol{R}^n)$, the Schwartz space of rapidly decreasing C^∞-functions, the Fourier transform of f is defined by

$$\hat{f}(\boldsymbol{\xi}) = \int_{\boldsymbol{R}^n} f(\boldsymbol{x}) e^{-2\pi i \boldsymbol{x}\cdot\boldsymbol{\xi}} d\boldsymbol{x} ,$$

where $\boldsymbol{x}\cdot\boldsymbol{\xi} = x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n$. Throughout this paper, we assume $f \in \mathscr{S}(\boldsymbol{R}^n)$ unless otherwise specified.

If $K(\boldsymbol{x}) = \Omega(\boldsymbol{x}')/|\boldsymbol{x}|^n$ is the Calderón-Zygmund kernel, then

$$\tilde{f}_\Omega(\boldsymbol{x}) = \lim_{\varepsilon\to 0} \int_{|\boldsymbol{y}|>\varepsilon} K(\boldsymbol{y}) f(\boldsymbol{x} - \boldsymbol{y}) d\boldsymbol{y}$$

exists almost everywhere and

$$\|\tilde{f}_\Omega\|_p \leqq A_p \|f\|_p \quad (1 < p < \infty) .$$

$\tilde{f}_\Omega$ is a conjugate integral in n-dimensions.

The spherical mean of order $\alpha > 0$ of f is

$$(1.1) \qquad (M_t^\alpha f)(\boldsymbol{x}) = c_\alpha t^{-n} \int_{|\boldsymbol{y}|<t} (1 - |\boldsymbol{y}|^2/t^2)^{\alpha-1} f(\boldsymbol{x} - \boldsymbol{y}) d\boldsymbol{y} ,$$

where $c_\alpha = \Gamma(\alpha + n/2)/\pi^{n/2}\Gamma(\alpha)$. Also we define

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