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SPECTRA OF MEASURES AS L_p MULTIPLIERS

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1. Preliminaries. Let G be a nondiscrete locally compact abelian group with the dual Γ, $M(G)$ the convolution measure algebra of finite regular Borel measures on G. For $\mu \in M(G)$, let $\|\mu\|$ denote the total variation norm, $\mu^1 = \mu$, $\mu^j = \mu^{j-1}*\mu$ $(j = 2, 3, \cdots)$, where $*$ denotes the convolution, $\hat{\mu}$ the Fourier-Stieltjes transform of, μ, and $\|\hat{\mu}\|_\infty = \sup\{|\hat{\mu}(\gamma)|; \gamma \in \Gamma\}$. We call μ a Hermitian measure if $\hat{\mu}(\gamma)$ is real valued on Γ. For $1 \leqq p \leqq \infty$, let $L_p(G)$ be the L_p space with respect to the Haar measure of G, $\| \quad \|_p$ the norm of $L_p(G)$. A bounded linear operator T on $L_p(G)$ is called an L_p multiplier if there exists $\hat{T} \in L_\infty(\Gamma)$ such that $T(f)\hat{} = \hat{T}\hat{f}$ for every $f \in L_p(G) \cap L_1(G)$. The set of all L_p multipliers will be written as $M_p(G)$ and the norm of $T \in M_p(G)$ is defined by

$$\|T\|_{M_p(G)} = \|T\|_{M_p} = \sup\{\|Tf\|_{L_p(G)};\ \|f\|_{L_p(G)} = 1\} .$$

Then $M_p(G)$ is a commutative Banach algebra with unit δ_0 as the convolution operator, where δ_0 is the Dirac measure with unit mass at $0 \in G$. Also for $T \in M_p(G)$, let $\tilde{T}$ be the Gelfand transform

$$\|\tilde{T}\|_{\Delta M_p} = \sup\{|h(T)|;\ h \text{ is a complex homomorphism on } \ M_p(G)\} ,$$

and Im $\tilde{T}$ the imaginary part of $\tilde{T}$.

Now it is known that any measure $\mu \in M(G)$ is contained in M_p(G) as a convolution operator, and $M_1(G)$ is isomorphic to $M(G)$, $M_2(G)$ to $L_\infty(\Gamma)$, $M_p(G)$ to $M_q(G)$ if $1/p + 1/q = 1$ $(1<p<\infty)$, and $M_1(G) \subseteqq M_p(G) \subseteqq M_2(G)$ $(1 \leqq p \leqq 2)$ (cf. [6]). For $T \in M_p(G)$, let $\mathrm{sp}(T, M_p)$ be the spectrum of T in $M_p(G)$, i.e., $\mathrm{sp}(T, M_p) = \{\lambda \in \boldsymbol{C};\ \lambda\delta_0 - T$ is not invertible in $M_p(G)\}$, where $\boldsymbol{C}$ is the complex plane. Then for $\mu \in M(G)$, we have $\mathrm{closure}(\hat{\mu}(\Gamma)) = \mathrm{sp}(\mu, M_2) \subseteqq \mathrm{sp}(\mu, M_p) \subseteqq \mathrm{sp}(\mu, M(G))$ $(1 \leqq p \leqq 2)$, where closure$(\hat{\mu}(\Gamma))$ is the closure of $\hat{\mu}(\Gamma)$ in the complex plane. Before stating our theorems, we make some preliminary comments. For $f \in L_1(G)$, it is well known and easy to show that $\mathrm{sp}(T_f, M_p(G)) = \hat{f}(\Gamma) \cup \{0\}$ for $1 \leqq p \leqq \infty$ if $T_f(g) = f*g$ for all $g \in L_p(G)$. However, since G is nondiscrete, the classical

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