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GLOBAL ASYMPTOTIC STABILITY IN A PERIODIC INTEGRODIFFERENTIAL SYSTEM

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A set of easily verifiable sufficient conditions are derived for the existence of a globally stable periodic solution in a system of nonlinear Volterra integrodifferential equations with periodic coefficients.

1. Introduction. The purpose of this article is to derive a set of "easily verifiable" sufficient conditions for the existence of a globally asymptotically stable strictly positive (componentwise) periodic solution of the integrodifferential system

$$\frac{dx_i(t)}{dt} = x_i(t)\left\{b_i(t) - a_{ii}(t)x_i(t) - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-u)x_j(u)du\right\}, \quad i = 1, 2, \cdots, n;\ t > t_0;\ t_0 \in (-\infty, \infty) \tag{1.1}$$

where b_i, a_{ij} $(i, j = 1, 2, \cdots, n)$ are continuous, positive periodic functions with a common period ω and K_{ij}: $[0, \infty) \to [0, \infty)$, $(i, j = 1, 2 \cdots, n; i \neq j)$ denote delay kernel about which more will be said below. In mathematical ecology (1.1) denotes a model of the dynamics of an n-species system in which each individual competes with all others of the system for a common pool of resources and the interspecific competition involves a time delay extending over the entire past as typified by the delay kernels K_{ij} in (1.1). The assumption of periodicity of the parameters b_i, a_{ij} $(i, j = 1, 2, \cdots, n)$ is a way of incorporating the periodicity of the environment (e.g. seasonal effects of weather, food supplies, mating habits etc.). We will need the following preparation.

LEMMA 1.1. *Assume that the delay kernels K_{ij} $(i, j = 1, 2, \cdots, n; i \neq j)$ are piecewise (locally) continuous such that the series $\sum_{r=0}^{\infty} K_{ij}(u + rw)$ converges uniformly with respect to u on $[0, \omega]$. Then any ω-periodic solution of (1.1) is also an ω-periodic solution of*

$$\frac{dx_i(t)}{dt} = x_i(t)\left\{b_i(t) - a_{ii}(t)x_i(t) - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}(t)\int_{t-\omega}^{t} H_{ij}(t-u)x_j(u)du\right\}, \quad i = 1, 2, \cdots, n, \tag{1.2}$$

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