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STIEFEL-WHITNEY HOMOLOGY CLASSES OF k-POINCARÉ-EULER SPACES

Dedicated to Professor Itiro Tamura on his sixtieth birthday

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1. Introduction and the statement of results. Let X be a polyhedron. It is said to be totally n-dimensional if there exists a locally finite triangulation K of X such that for each $\sigma \in K$, an n-dimensional simplex τ exists in K satisfying $\sigma \prec \tau$ or $\sigma = \tau$. (See Akin [1].) A totally n-dimensional polyhedron X is an n-dimensional k-Euler space if there exist a locally finite triangulation K of X and a subcomplex L of K satisfying the following:

(1) $|L|$ is a totally $(n-1)$-dimensional polyhedron or empty.

(2) The cardinality of $\{\tau \in K \mid \sigma \prec \tau\}$ is even for every σ in $K - L$, whenever $\dim \sigma \geqq n - k$.

(3) The cardinality of $\{\tau \in K \mid \sigma \prec \tau\}$ is odd for every σ in L, whenever $\dim \sigma \geqq n - k$.

(4) The cardinality of $\{\tau \in L \mid \sigma \prec \tau\}$ is even for every σ in L, whenever $\dim \sigma \geqq n - k - 1$.

We usually denote ∂X instead of $|L|$. If X is an n-dimensional k-Euler space, then ∂X clearly is an $(n-1)$-dimensional k-Euler space. An n-dimensional k-Euler space X is closed if X is compact and ∂X is empty. If $k \geqq n$, we said n-dimensional k-Euler spaces to be n-dimensional $\boldsymbol{Z}_2$-Euler spaces. (See [10].)

Let X be an n-dimensional k-Euler space with a triangulation K. Then the i-th Stiefel-Whitney homology class $s_i(X)$ in $H_i^{\inf}(X, \partial X; \boldsymbol{Z}_2)$ is the homology class determined as the i-skeleton $\bar{K}^i$ of the first barycentric subdivision $\bar{K}$ of K for $n - k < i \leqq n$. Here $H_*^{\inf}$ is the homology theory of infinite chains. The Stiefel-Whitney homology classes of k-Euler spaces are well defined by Proposition 2.2.

Since an n-dimensional differentiable manifold M has a triangulation, the i-th Stiefel-Whitney homology class $s_i(M)$ can be defined as above for $0 \leqq i \leqq n$. Whitney [16] announced that the i-th Stiefel-Whitney homology class of an n-dimensional differentiable manifold M is the Poincaré dual of the $(n-i)$-th Stiefel-Whitney class $w^{n-i}(M)$. Its proof was outlined