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SUBMANIFOLDS OF THE GRASSMANN MANIFOLD WITH VANISHING CHERN FORMS

Dedicated to the memory of Professor Yozo Matsushima

PATRICK COULTON

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Introduction. We shall adopt the notation of Griffiths [2] throughout this paper. A holomorphic mapping between manifolds induces a holomorphic foliation on the domain manifold. In particular, for a complex manifold M^n immersed in $\boldsymbol{C}^N$, we have

$$\Gamma(n, N)\colon M^n \dashrightarrow G(n, N) ,$$

where $\Gamma=\Gamma(n, N)$ is the generalized Gauss map which is a holomorphic mapping and Γ_* has constant rank off a set of measure zero. Thus, $\ker(\Gamma_*)$ is an integrable subbundle almost everywhere. In fact, if E denotes the universal bundle of $G(n, N)$, then $\Gamma^*(E) = TM$, and the Chern forms on E pullback to the Chern forms on TM. Therefore, we are interested in studying the Chern forms on the universal bundle along submanifolds of $G(n, N)$.

In this paper we will give conditions in terms of the Chern forms on the universal bundle of the Grassmannian which imply that a submanifold of sufficiently high dimension has a parallel subbundle in the universal bundle.

Let $c_j(\Omega_E|_M)$ denote the j-th Chern form on the universal bundle of $G(n, N)$ restricted to M. Let $c_1(\Omega_q|_M)$ denote the first Chern form on $\boldsymbol{CP}^q$.

We shall prove the following:

THEOREM 2.8. *Assume that M^r is a complex submanifold of $G(n, N)$, with $r \geqq 2$. If $c_k(\Omega_E|_M) \neq 0$, and $c_{k+1}(\Omega_E|_M) = 0$, for*

$$k \leqq (r + N - n - 2)/(N - n) ,$$

then there exists a constant $(n - k)$-subbundle of $E|_M$.

The following Corollary is a direct result of the Theorem.

COROLLARY 2.9. *Assume that M^r is a submanifold of $G(n, N)$, with $r \geqq 2$, and $c_2(\Omega_E|_M) = 0$. Then M is contained in $G(1, N - n + 1) \subset G(n, N)$.*