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A DIMENSION FORMULA FOR A CERTAIN SPACE OF AUTOMORPHIC FORMS OF $SU(p, 1)$, II: THE CASE OF $\Gamma(N)$ WITH $N \geqq 3$

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0. Introduction. In the previous paper [5], we derived a dimension formula for the spaces of cusp forms of $SU(p, 1)$ in a closed form in the case of neat lattices in $SU(p, 1)$. With the use of this formula, we shall give, in the present paper, more explicit expressions for such dimensions in the case of the congruence subgroups $\Gamma(N)$ with $N \geqq 3$ in terms of the arithmetic quantities.

For $SU(2, 1)$, explicit description of such dimensions was given by Cohn [3] for $\Gamma(1)$ defined for the base field $\boldsymbol{Q}(\sqrt{-1})$. There he calculated the volume of $\Gamma(1)\backslash SU(2, 1)$ and explained in detail how elliptic elements contribute to the dimension formula. On the other hand, for $SU(p, 1)$ Zeltinger has calculated the volume of $\Gamma(1)\backslash SU(p, 1)$ in [12]. Thus, in our case, in view of the result in [5] (Theorem 1.1 in this paper), we have only to describe in terms of the arithmetic quantities the contribution of unipotent elements to the dimension formula. We shall obtain such a description in this paper.

In § 1, we shall recall the definitions and the results in [5] and state the main theorem in this paper. In § 2, we explain the relation between certain quantities related to the $\Gamma(1)$-inequivalent cusps and the theory of adele groups and investigate the adelized group $SU(p, 1)_A$, following the method of Arakawa [1, § 3]. We also give another proof of the result concerning the number of $\Gamma(1)$-inequivalent cusps obtained in Zeltinger [12]. (By a similar method, one can also prove a more general result concerning $SU(p, q)$, conjectured by Zeltinger. See Corollary 2.7.) The third section is devoted to a proof of the main theorem.

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NOTATION. We denote by $\boldsymbol{C}$, $\boldsymbol{R}$, $\boldsymbol{Q}$, $\boldsymbol{Z}$ and $\boldsymbol{N}$, respectively, the field of complex numbers, the field of real numbers, the field of rational numbers, the ring of rational integers and the set consisting of all natural