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CHARACTERIZATION OF QUASI-DISKS AND TEICHMÜLLER SPACES

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1. Introduction and main results. A simply connected domain in the Riemann sphere $\hat{C}$ is called a *quasi-disk* if it is the image of the unit disk by a quasiconformal automorphism of the sphere. Since Ahlfors' investigation [2] in 1963, several characteristic properties of quasi-disks have been studied by many authors. As a result, quasi-disks are related to various topics in analysis. A bird's eye view of these studies are given in Gehring [9]. Among them, the topics with which we are concerned in this article are the BMO *extension property* and the *Schwarzian derivative property*.

Let W be a domain in $\boldsymbol{C}$. Then $f \in L^1_{\mathrm{loc}}(W)$ belongs to BMO(W) if

$$\|f\|_{*,W} = \sup_{B \subset W} \frac{1}{|B|} \int_B |f - f_B| dxdy < +\infty ,$$

where B is a disk in W with $\bar{B} \subset W$, $|B| = \int_B dxdy$ and $f_B = |B|^{-1} \int_B fdxdy$.

Let $\mathscr{F}$ be a subclass of BMO(W). We say that W has the BMO *extension property for* $\mathscr{F}$ if there exists a constant $C_1 > 0$ such that for every $f \in \mathscr{F}$ there is an $F \in \mathrm{BMO}(\boldsymbol{C})$ with $F|W = f$ and

$$\|F\|_{*,C} \leqq C_1 \|f\|_{*,W} . \tag{1.1}$$

Jones [11] has shown that a simply connected domain $\varDelta$ ($\neq \boldsymbol{C}$) in $\boldsymbol{C}$ is a quasi-disk if and only if $\varDelta$ has the BMO extension property for BMO($\varDelta$) (see also Gehring [9]).

In the first part, we shall strengthen the "if" part of Jones' result.

THEOREM 1. *Let $\varDelta$ ($\neq \boldsymbol{C}$) be a simply connected domain in $\boldsymbol{C}$. If $\varDelta$ has the* BMO *extension property for* ABD($\varDelta$), *then $\varDelta$ is a quasi-disk, where* ABD($\varDelta$) *is the space of all bounded holomorphic functions in $\varDelta$ with finite Dirichlet integrals.*

In the second part, we shall investigate *Teichmüller spaces* of Fuchsian groups and the *Schwarzian derivative property*, independently

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