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ASYMPTOTIC ESTIMATES FOR MODULI OF EXTREMAL RINGS

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Abstract. For $n \geqq 2$ and $0 < a < 1$ let $R_n(a)$ denote the extremal ring domain consisting of the unit ball in n-space minus the closed slit $[-a, a]$ along the x_1-axis. Significant lower and upper limits as n tends to ∞ are obtained for the expressions

$$\operatorname{mod} R_n(a) - n + \frac{1}{2} \log n$$

and

$$n^{1/2-n} \operatorname{mod} R_n(a)^n ,$$

where mod denotes the conformal modulus.

1. Introduction. In this paper we find asymptotic lower and upper limits as n tends to ∞ for the modulus of certain extremal rings in n-space.

For $n \geqq 2$ and $0 < a < 1$ we let $R = R_n(a)$ denote the ring in $\boldsymbol{R}^n$ consisting of the open unit ball B^n minus the closed slit $[-a, a]$ along the x_1-axis. The *conformal capacity* of R is defined to be

$$\operatorname{cap} R = \inf_u \int_R |\nabla u|^n d\omega ,$$

where $u \in C^1(R)$, $u = 0$ on the slit $[-a, a]$, and $u = 1$ on the boundary sphere S^{n-1}. The *modulus* of R is defined by

$$\operatorname{mod} R = (\sigma_{n-1}/\operatorname{cap} R)^{1/(n-1)} , \qquad \sigma_{n-1} = m_{n-1}(S^{n-1}) .$$

The rings $R_n(a)$ are *extremal* in the following sense: If R is any ring in $\boldsymbol{R}^n$ consisting of the unit ball minus a continuum whose diameter is at least $2a$, then $\operatorname{mod} R \leqq \operatorname{mod} R_n(a)$ (cf. [An1]). This extremal property of the rings $R_n(a)$ makes them useful in the study of the distortion properties of quasiconformal mappings in n-space (cf. [G], [AVV]), and we therefore wish to obtain all possible information about these rings.

The asymptotic behavior of $R_n(a)$ has been studied as a tends to 0 and to 1 and as n tends to ∞. In particular, it has been shown [An2, Theorem 2, p. 7] that for each a, $0 < a < 1$,

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