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THE FIRST EIGENVALUE OF HOMOGENEOUS MINIMAL HYPERSURFACES IN A UNIT SPHERE $S^{n+1}(1)$

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1. Introduction. It is well known as a Theorem of Takahashi [8] that a Riemannian n manifold M immersed into an $(n+1)$-dimensional unit sphere $S^{n+1}(1)$ is minimal if and only if each coordinate function is an eigenfunction of Δ on M with eigenvalue n. This implies that the first eigenvalue of M is not greater than n.

Ogiue [12] and Yau [11] independently posed the following problem: "What kind of compact embedded minimal hypersurfaces of $S^{n+1}(1)$ do satisfy the condition that the first eigenvalue is just n?"

It is difficult in general to compute eigenvalues in practice. In [4] a little more restricted problem is considered, that is, they compute the first eigenvalues for some of the compact homogeneous minimal hypersurfaces of $S^{n+1}(1)$. There are 14 kinds of compact homogeneous minimal hypersurfaces of $S^{n+1}(1)$ (cf. Hsiang and Lawson [1]), and some of them are left untouched. We note that a homogeneous hypersurface of $S^{n+1}(1)$ has constant principal curvatures so that it is isoparametric.

The purpose of this paper is to compute the first eigenvalues for some of them and prove the following.

THEOREM. *If M is an n-dimensional compact homogeneous minimal hypersurface in a unit sphere with r distinct principal curvatures, then the first eigenvalue of the Laplacian on M is n unless $r = 4$.*

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2. Laplacian of homogeneous hypersurfaces in $S^{n+1}(1)$. Hsiang and Lawson [1] proved that every compact homogeneous hypersurface in $S^{n+1}(1)$ can be obtained as follows.

Let (G, K) be a symmetric pair of compact type of rank 2 with bi-invariant Riemannian metric $\hat{g}$ induced from the Killing form B_G of the Lie algebra $\mathfrak{g}$ of G. Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be the Cartan decomposition associated with (G, K). We regard $\mathfrak{p}$ as a Euclidean space with inner product $-B_G$. Choose a maximal Abelian subspace $\mathfrak{a}$ in $\mathfrak{p}$ and denote by Σ the set of all roots of $\mathfrak{g}$. Let Σ_+ be the set of all positive elements in Σ with