Tôhoku Math. Journ.
37 (1985), 489-511.

STABILITY BY DECOMPOSITIONS FOR VOLTERRA EQUATIONS

THEODORE ALLEN BURTON AND WADI ELIAS MAHFOUD

(Received September 19, 1984)

Abstract. We consider a system of integrodifferential equations of the form

$$x' = A(t)x + \int_0^t C(t, s)x(s)ds \tag{1}$$

which we then write as

$$x' = L(t)x + \int_0^t C_1(t, s)x(s)ds + \left(\frac{d}{dt}\right)\int_0^t H(t, s)x(s)ds\ . \tag{2}$$

A number of Lyapunov functionals are constructed for (2) yielding necessary and sufficient conditions for stability of the zero solution of (1).

1. Introduction.

We consider the system

$$x' = A(t)x + \int_0^t C(t, s)x(s)ds \tag{1.1}$$

in which $A(t)$ is an $n\times n$ matrix continuous for $0 \leqq t < \infty$, $C(t, s)$ is an $n\times n$ matrix continuous for $0 \leqq s \leqq t < \infty$, and $n \geqq 1$.

We write (1.1) as

$$x' = L(t)x + \int_0^t C_1(t, s)x(s)ds + \frac{d}{dt}\int_0^t H(t, s)x(s)ds \tag{1.2}$$

and discuss stability and instability of the zero solution of (1.1) via the construction of Lyapunov functionals for the system (1.2).

Evidently, (1.1) can be regarded as a special case of (1.2) and therefore any stability result for (1.2) is also a stability result for (1.1). However, the most interesting stability results of this paper are those obtained by converting (1.1) to (1.2). It turns out that (1.1) can be reduced to (1.2) in several ways and consequently a variety of stability results will be obtained. In most cases, we obtain simple and practical results under mild conditions.

The following terminology is used throughout this paper. For any $t_0 \geqq 0$ and any continuous function ϕ: $[0, t_0] \to R^n$, a solution of (1.2) and hence of (1.1) is a continuous function x: $[0, \infty) \to R^n$, denoted by $x(t, t_0, \phi)$ or $x(t)$, which satisfies (1.2) for $t \geqq t_0$ and such that $x(t) = \phi(t)$ for $0 \leqq t \leqq t_0$. The solution $x = 0$ is called the zero solution.