Tôhoku Math. Journ.
37 (1985), 455-462.

# STABILITY PROPERTIES OF SOLUTIONS OF LINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

WANG ZHICHENG, LI ZHIXIANG AND WU JIANHONG

(Received August 2, 1984)

Consider the following systems of Volterra equations

$$Z'(t) = A(t)Z(t) + \int_0^t C(t, s)Z(s)ds , \tag{1}$$

$$y'(t) = A(t)y(t) + \int_0^t C(t, s)y(s)ds + f(t) , \tag{2}$$

$$x'(t) = A(t)x(t) + \int_{-\infty}^t C(t, s)x(s)ds + f(t) , \tag{3}$$

where $A$ is an $n \times n$ matrix of functions continuous on $(-\infty, +\infty)$, $C$ is an $n \times n$ matrix of functions continuous for $-\infty < s \leqq t < \infty$, and $f: (-\infty, +\infty) \to R^n$ is continuous. For the fundamental properties of solutions of these equations, we refer to Driver [4] and Burton [2]. Some of those properties may be listed as follows:

(a) There is an $n \times n$ matrix $Z(t)$ satisfying (1) on $[0, \infty)$ and $Z(0) = I$. For each $z_0 \in R^n$, there is a unique solution $z(t, 0, z_0)$ of (1) on $[0, \infty)$ and $z(t, 0, z_0) = Z(t)z_0$.

(b) For (2), given $t_0 \geqq 0$ and a continuous function $\varphi: [0, t_0] \to R^n$, there is a unique solution $y(t, t_0, \varphi)$ satisfying (2) on $[t_0, \infty)$ with $y(t, t_0, \varphi) = \varphi(t)$ for $t \in [0, t_0]$.

(c) For (3) we suppose that $\int_{-\infty}^0 |C(t, s)| ds$ is continuous for $0 \leqq t < \infty$. If $t_0 \in R$ and if $\varphi: (-\infty, t_0] \to R^n$ is a bounded continuous function, there is a unique solution $x(t, t_0, \varphi)$ satisfying (3) on $[t_0, \infty)$ with $x(t, t_0, \varphi) = \varphi(t)$ for $t \leqq t_0$.

(d) There is a unique $n \times n$ matrix $R(t, s)$ satisfying

$$\frac{\partial}{\partial s}R(t, s) = -R(t, s)A(s) - \int_s^t R(t, u)C(u, s)du , \qquad R(t, t) = I \tag{4}$$

for $0 \leqq s \leqq t < \infty$. For each $y_0 \in R^n$, the unique solution $y(t, 0, y_0)$ of (2) satisfies

$$y(t, 0, y_0) = Z(t)y_0 + \int_0^t R(t, s)f(s)ds , \tag{5}$$