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ON FULL SUBGROUPS OF CHEVALLEY GROUPS*

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Introduction. Let G be a split algebraic absolutely almost simple group defined over a field k. For a split maximal k-subtorus T of G let $\Sigma = \Sigma(G, T)$ denote the root system of G with respect to T. Let $\{x_\varepsilon, \varepsilon \in \Sigma\}$ be a system of isomorphisms, normalized as usual (see, for example, Steinberg [4]), from the additive group onto the root subgroups with respect to T.

We say (in the spirit of O'Meara [2, 3]) that a subgroup H of $G(k)$ is *full* if for every g in $G(k)$ and ε in Σ there exists a non-zero $c = c(g, \varepsilon)$ in k such that $g^{-1}x_\varepsilon(c)g \in H$. Thus, H is full if and only if its intersection with any root subgroup (relative to any maximal split k-torus) contains at least two elements.

For a subset R of k we denote by $G^E(R)$ the subgroup of $G(k)$ generated by all $x_\varepsilon(a)$, where $\varepsilon \in \Sigma$ and $a \in R$. Here "E" stands for "elementary".

A subset R of k is called *full* (cf., Vaserstein [7]) if for every y in k there is a non-zero r in R such that $yr \in R$. For a subring R it means that k is its field of fractions. Note that in this paper a ring is not required to have identity.

The results of the present paper are modeled on the results of Vaserstein [7], the methods are also similar. However the situation for groups of type C_n in characteristic 2 turns out to be more complicated.

We assume throughout (except in the last section) that the rank of G is greater than one. If $\mathrm{rank}(G) = 1$, i.e., G is of type A_1, then the conclusions of Theorems 1-5 below are false, see [7] and the last section, where we also discuss possible generalizations of our results.

The following Theorems 1-5 summarize our main results. More precise and detailed statements are given in the corresponding sections.

THEOREM 1. *For every full subring R of k, the subgroup $G^E(R)$ of $G(k)$ is full.*

THEOREM 2. ("Arithmeticity Theorem"). *Every full subgroup H of $G(k)$ contains $G^E(A)$ for some full subring A of k with the exception of*

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