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ROTATIONAL SURFACES IN A PSEUDO-RIEMANNIAN 3-SPHERE

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1. Introduction. K. Akutagawa has recently shown the following interesting result.

THEOREM A. *Let $S_1^{n+1}(c)$ be a pseudo-Riemannian $(n+1)$-sphere of signature $(1, n)$ and of constant positive sectional curvature c. Let M be a complete, space-like hypersurface with constant mean curvature h in $S_1^{n+1}(c)$. If*

(i) $|h| \leqq c^{1/2}$ *when* $n = 2$,

(ii) $|h| < (2/n)[(n-1)c]^{1/2}$ *when* $n \geqq 3$,

then M is totally umbilical.

In this paper, we shall show in case $n = 2$ that the estimate in Theorem A is sharp. In fact, for each constant $h > 1$ we shall construct some families of complete, space-like, rotational surfaces in S_1^3 $(:= S_1^3(1))$ with constant mean curvature h, none of which are umbilical.

(Added on March 6, 1985). K. Akutagawa has kindly sent us his preprint [1] in which he proves the above Theorem A and also independently shows that the estimate in (i) is sharp.

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2. Statement of results. All the surfaces in the following Theorems 1, 2 and 3 except those in Theorem 3 (iii) turn out not to be umbilical by Proposition 1 in Section 3 and (4.7) in Section 4. We refer the readers to Section 3 for the terminology.

THEOREM 1. (Spherical rotational, space-like surfaces). *Let h be a constant, $h > 1$.*

(i) *For each constant $a > (h^2-1)^{1/2}/2$, we define the function $u(s)$ by*

$$u(s) = [ah + \{a^2 - (h^2-1)/4\}^{1/2} \cosh 2(h^2-1)^{1/2}s]/2(h^2-1)\,,$$

$s \in R$, and the functions $\phi(s)$, $x_1(s)$, $x_3(s)$ and $x_4(s)$ by