

A UNICITY THEOREM FOR MEROMORPHIC MAPS OF A COMPLETE KÄHLER MANIFOLD INTO $P^N(C)$

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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1. Introduction. In 1926, R. Nevanlinna proved the following unicity theorem for meromorphic functions on C ([12]).

THEOREM. *Let ϕ, ψ be nonconstant meromorphic functions on C . If there exist five distinct values a_1, \dots, a_5 such that $\phi^{-1}(a_i) = \psi^{-1}(a_i)$ ($1 \leq i \leq 5$), then $\phi \equiv \psi$.*

The author gave several types of generalizations of this to the case of meromorphic maps of C^n into $P^N(C)$ in his papers [3] ~ [8]. In this paper, we study meromorphic maps of an n -dimensional complete Kähler manifold M into $P^N(C)$ and give a new type of unicity theorem in the case where the universal covering of M is biholomorphic to the ball in C^n and meromorphic maps satisfy a certain growth condition.

Let M be an n -dimensional connected Kähler manifold with Kähler form ω and f be a meromorphic map of M into $P^N(C)$. For $\rho \geq 0$ we say that f satisfies the condition (C_ρ) if there exists a nonzero bounded continuous real-valued function h on M such that

$$\rho \Omega_f + dd^c \log h^2 \geq \text{Ric } \omega,$$

where Ω_f denotes the pull-back of the Fubini-Study metric form on $P^N(C)$ by f and $d^c = (\sqrt{-1}/4\pi)(\bar{\partial} - \partial)$.

Take a point $p \in M$. We represent f as $f = (f_1 : \dots : f_{N+1})$ on a neighborhood of p with holomorphic functions f_i , where $\mathbf{f} := (f_1, \dots, f_{N+1}) \neq (0, \dots, 0)$. Let \mathcal{M}_p denote the field of all germs of meromorphic functions at p . For each $k \geq 0$ we consider the \mathcal{M}_p -submodule \mathcal{F}_p^k of \mathcal{M}_p^{N+1} generated by all elements $(\partial^{|\alpha|} / \partial z^\alpha) \mathbf{f}$ with $|\alpha| \leq k$, where $z = (z_1, \dots, z_n)$ is a system of holomorphic local coordinates around p and $|\alpha| = \alpha_1 + \dots + \alpha_n$ for $\alpha = (\alpha_1, \dots, \alpha_n)$.

By definition, the k -th rank of f is given by

$$r_f(k) := \text{rank}_{\mathcal{M}_p} \mathcal{F}_p^k - \text{rank}_{\mathcal{M}_p} \mathcal{F}_p^{k-1},$$

which does not depend on the choices of a point $p \in M$, a reduced