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SUBMANIFOLDS WITH PROPER d-PLANAR GEODESICS IMMERSED IN COMPLEX PROJECTIVE SPACES

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Introduction. Recently, several authors studied submaifolds with "simple" geodesics immersed in space forms. For example, planar geodesic immersions were studied in [6], [8], [13], [14], geodesic normal sections in [3] and helical immersions in [15]. In [9], Nakagawa also introduced a notion of cubic geodesic immersions. Let M and $\tilde{M}$ be connected complete Riemannian manifolds of dimensions n and $n+p$, respectively. An isometric immersion ι of M into $\tilde{M}$ is called a *d-planar geodesic immersion* if each geodesic in M is mapped locally under ι into a d-dimesional totally geodesic submanifold of $\tilde{M}$. In particular, if a 3-planar geodesic immersion is isotropic, then it is called a *cubic geodesic immersion.* In this paper, we study a proper d-planar geodesic Kählerian immersion $\iota: M \to \boldsymbol{CP}^m(c)$ of a Kähler manifold M into a complex projective space $\boldsymbol{CP}^m(c)$ of constant holomorphic sectional curvature c and proper cubic geodesic totally real immersion $\iota: M \to \boldsymbol{CP}^m(c)$ of a Riemannian manifold M, where "proper" means that the image of each geodesic in M is not $(d-1)$-planar. Here and elsewhere, m in N^m denotes the complex dimension, if N is a complex manifold.

In a complex projective space $\boldsymbol{CP}^m(c)$ of complex dimension m, an odd-dimensional totally geodesic submanifold is a totally real submanifold $\boldsymbol{RP}^l(c/4)$ of constant sectional curvature $c/4$. In § 2 we show that if $\iota: M^n \to \boldsymbol{CP}^m(c)$ is a proper d-planar geodesic Kählerian immersion of a Kähler manifold M^n and d is odd, then $M^n = \boldsymbol{CP}^n(c/d)$ and ι is equivalent to the d-th Veronese map. Here we recall the definition of k-th Veronese map ($k = 1, 2, \cdots$). This is a Kähler imbedding $\boldsymbol{CP}^n(c/k) \to \boldsymbol{CP}^{m'}(c)$ given by

$$[z_i]_{0 \leqq i \leqq n} \mapsto \left[\left(\frac{k!}{k_0! \cdots k_n!}\right)^{1/2} z_0^{k_0} \cdots z_n^{k_n}\right]_{k_0+\cdots+k_n=k},$$

where $[*]$ means the point of the projective space with the homogeneous coordinates $*$ and $m' = \binom{n+k}{k} - 1$. More generally, we prove that if

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