Tôhoku Math. Journ.
38 (1986), 281–295.

CONTINUITY OF CERTAIN DIFFERENTIALS ON FINITELY AUGMENTED TEICHMÜLLER SPACES AND VARIATIONAL FORMULAS OF SCHIFFER-SPENCER'S TYPE

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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(Received May 17, 1985)

1. Introduction and statement of results. For a Riemann surface R^*, the finitely augmented Teichmüller space $\hat{T}(R^*)$ of R^* is the set of all marked Riemann surfaces R with at most a finite number of nodes such that there is a marking-preserving deformation of R^* to R, and $\hat{T}(R^*)$ is equipped with the conformal topology. (For the details, see [7, §1, 1°)].) Here we recall some of definitions.

For two given points R_1 and R_2 in $\hat{T}(R^*)$, a marking-preserving deformation $(f; R_1, R_2)$ of R_1 to R_2 is a marking-preserving continuous surjection f from R_1 onto R_2 such that f^{-1} restricted to $R_2 - \bar{U}$ is quasiconformal for every neighborhood U of the set $N(R_2)$ on R_2, and that $f^{-1}(p)$ is either a node of R_1 or a simple closed curve on $R_1 - N(R_1)$ for every p in $N(R_2)$, where here and in the sequel, $N(R)$ means the set of all nodes of R. A one-parameter family $\{(f_t; R_t, R_0)\}_{t\in(0,1]}$ of marking-preserving deformations f_t of $R_t \in \hat{T}(R^*)$ to $R_0 \in \hat{T}(R^*)$ is called *admissible* if

$$\lim_{t\to 0} K(f_t^{-1}, R_0 - \bar{U}) = 1$$

for every neighborhood U of $N(R_0)$, where here and in the sequel $K(f, E)$ is the maximal dilatation of a quasiconformal mapping f on a Borel set E. Recall that R_t converges to R_0 in $\hat{T}(R^*)$ if and only if there is an admissible family $\{(f_t; R_t, R_0)\}$.

In [7, §3], certain continuity property of holomorphic and harmonic differentials on $\hat{T}(R^*)$ was investigated. In particular, we showed strongly metrical continuity of period reproducers on $\hat{T}(R^*)$. Namely, let $\sigma(c, R)$ be the period reproducer for a 1-cycle c on $R \in \hat{T}(R^*)$ in the space $\Gamma_h(R)$ of all square integrable (real) harmonic differentials on $R - N(R)$ (cf. [7, §1, 2°)]). Then we have the following:

THEOREM A ([7, Proposition 4]). *Let an admissible family $\{(f_t; R_t, R_0)\}_{t\in(0,1]}$ of marking-preserving deformations and a 1-cycle d on*