Tôhoku Math. Journ.
38 (1986), 269-279.

INFINITESIMAL DEFORMATIONS OF TSUCHIHASHI'S CUSP SINGULARITIES

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(Received April 25, 1985)

0. Introduction. Let (X, x_0) be a normal isolated singularity of dimension n. Assume that X is a Stein neighborhood of the singular point x_0 and that x_0 is the only singularity of X. The set of first order infinitesimal deformations of X is the finite dimensional vector space T_X^1, which is isomorphic to $\mathrm{Ext}^1_{O_X}(\Omega^1_X, O_X)$. In [FK] Freitag and Kiehl proved that Hilbert modular cusp singularities of dimension greater than two are rigid in the sense of Schlessinger [Sc], that is, $T_X^1 = 0$. Behnke and Nakamura computed T_X^1 for two dimensional cusp singularities ([B 1], [B 2] and [N]).

Here we are interested in deformations of normal isolated singularities constructed by Tsuchihashi in [T]. Theorem 3 shows that these singularities of dimension three are not rigid in general.

Thanks are due to Professors T. Oda, M. Namba and M.-N. Ishida for many helpful conversations.

1. Tsuchihashi's cusp singularities. Let N be a free $\boldsymbol{Z}$-module of rank $n > 1$ and $N_{\boldsymbol{R}} := N \otimes_{\boldsymbol{Z}} \boldsymbol{R}$. Consider a pair (C, Γ) consisting of a nondegenerate open convex cone C in $N_{\boldsymbol{R}}$ and a subgroup Γ in $GL(N) := \mathrm{Aut}_{\boldsymbol{Z}}(N)$ satisfying the following conditions:

(i) C is Γ-invariant.

(ii) The action of Γ on $D := C/\boldsymbol{R}_{>0}$ is properly discontinuous and fixed point free.

(iii) The quotient space D/Γ is compact.

In [T] Tsuchihashi has associated to such a pair (C, Γ) an isolated singularity, which we may call Tsuchihashi's cusp singularity. This is a singular point of the normal analytic space $X := ((N_{\boldsymbol{R}} + \sqrt{-1}C)/N \cdot \Gamma) \cup \{x_0\}$.

Let N^* be the dual $\boldsymbol{Z}$-module of N with the natural pairing $\langle\ ,\ \rangle$: $N^* \times N \to \boldsymbol{Z}$, and let dy and dy' the Lebesgue measures on $N_{\boldsymbol{R}}$ and $N^*_{\boldsymbol{R}}$ respectively. Let $C^* := \{y' \in N^*_{\boldsymbol{R}};\ \langle y', y\rangle > 0$ for all y in $\bar{C} \setminus \{0\}\}$ be the dual cone of C. The characteristic function of the cone C is

$$\phi_C(y) := \int_{C^*} \exp(-\langle y', y\rangle) dy'$$