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STABILITY OF HARMONIC MAPS AND STANDARD MINIMAL IMMERSIONS

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1. Introduction. Let f be a smooth map of a compact Riemannian manifold M into another Riemannian manifold N. The *energy functional* $E(f)$ for f is defined by

$$E(f) = (1/2) \int_M \| df \|^2 dv_M \ .$$

A smooth map f of M into N is called a *harmonic map* if f is a critical point of the energy functional E. A harmonic map f is called *stable* if every second variation of E at f is nonnegative. Let S^n be an n-dimensional Euclidean sphere. Then the following remarkable theorems are known.

THEOREM (Xin [22]). *For $n \geqq 3$ there exists no nonconstant stable harmonic map from S^n to any Riemannian manifold.*

THEOREM (Leung [5]). *For $n \geqq 3$ there exists no nonconstant stable harmonic map from any compact Riemannian manifold to S^n.*

It is natural to ask what kind of a compact Riemannian manifold M has the property that there exists no nonconstant stable harmonic map from M to any Riemannian manifold nor from any compact Riemannian manifold to M. We call such an M *harmonically unstable*. We know topological restrictions on harmonically unstable Riemannian manifolds; if M is harmonically unstable, then by a classical result on closed geodesics we have $\pi_1(M) = \{1\}$ and by the theorem of Sacks and Uhlenbeck [15] $\pi_2(M) = \{1\}$.

The purpose of this note is to classify harmonically unstable compact symmetric spaces.

THEOREM 1. *A compact symmetric space M is harmonically unstable, if and only if M is a product of simply connected compact irreducible symmetric spaces belonging to the following list;*

(i) *simple Lie groups of type A_n $(n \geqq 2)$, B_2 and C_n $(n \geqq 3)$,*

(ii) *$SU(2n)/Sp(n)$ $(n \geqq 3)$,*

(iii) *spheres S^n $(n \geqq 3)$,*