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CLASS NUMBERS OF QUADRATIC EXTENSIONS OF ALGEBRAIC NUMBER FIELDS

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Introduction. For a number field K, denote by C_K the ideal class group of K. Let n be a given natural number greater than 1. In [5], Nagell proved that there exist infinitely many imaginary quadratic fields with class numbers divisible by n. The corresponding result for real quadratic fields was obtained by Yamamoto [11] and Weinberger [10]. In the same paper, Yamamoto constructed infinitely many imaginary quadratic fields K such that C_K contains a subgroup isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^2$. These results were recently generalized for non totally real fields of arbitrary degrees by Azuhata-Ichimura [1], and for totally real fields of arbitrary degrees by Nakano [7]. To be more precise, they constructed, for any integers $m, n > 1$ and $r_1, r_2 \geqq 0$ with $r_1 + 2r_2 = m$, infinitely many number fields K of degree m with just r_1 real primes such that C_K contains a subgroup isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^{r_2+1}$.

The main purpose of this paper is to prove certain relative versions of the above results. In this direction, Naito obtained a generalization of Yamamoto's result on imaginary quadratic fields. He constructed in [6], for a given totally real field F, infinitely many totally imaginary quadratic extensions K/F such that C_K contains a subgroup H isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^2$ with $H \cap C_F = 1$. On the other hand, we obtain a generalization of Yamamoto's result on real quadratic fields (Theorem 1). Our second result is an analogue of Nakano's result over quadratic fields (Theorem 2).

For $n = 3, 5$ or 7, it was known that there exist infinitely many real quadratic fields K such that C_K contains a subgroup isomorphic to $(\boldsymbol{Z}/n\boldsymbol{Z})^2$ (for $n = 3$ by Yamamoto [11, Part II], for $n = 5$ or 7 by Mestre [4]). We note that a stronger result for $n = 3$ was obtained by Craig [2]. Our third result is a relative version of the above result for $n = 3$ (Theorem 3).

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Statement of the results.

THEOREM 1. *Let F be a number field of finite degree with $r_2 = 0$ or 1, where r_2 is the number of imaginary primes of F. Then for any*