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SOME REMARKS ON MEAN-VALUES OF SUBHARMONIC FUNCTIONS

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0. Introduction and notation. This paper is in two loosely related parts: the first part gives conditions for a nonnegative continuous function or its logarithm to be subharmonic, and the second includes a Fejér-Riesz type theorem for subharmonic functions.

The open ball, the closed ball, and the sphere of centre x and radius r in $\boldsymbol{R}^n$ $(n \geqq 2)$ are denoted by $B(x, r)$, $\bar{B}(x, r)$ and $S(x, r)$. We denote n-dimensional Lebesgue measure by ω and $(n-1)$-dimensional surface area measure on $S(x, r)$ by σ, and we write $\Omega(r)$ for the volume of $B(x, r)$ and $\Sigma(r)$ for the surface-area of $S(x, r)$. If a function f, defined at least on $\bar{B}(x, r)$, is ω-integrable on $B(x, r)$ and σ-integrable on $S(x, r)$, we define means as follows:

$$A(f, x, r) = (\Omega(r))^{-1}\int_{B(x,r)} f d\omega$$

and

$$M(f, x, r) = (\Sigma(r))^{-1}\int_{S(x,r)} f d\sigma .$$

Throughout the paper G will be a nonempty open subset of $\boldsymbol{R}^n$. Recall that a function is hypoharmonic in G if and only if in each connected component of G it is either subharmonic or identically $-\infty$. We shall say that a function is PL if its logarithm is hypoharmonic in G.

1. Mean value conditions for subharmonicity.

1.1. The following results are well-known.

THEOREM A. *Let $u: G \to \boldsymbol{R}$ be continuous in G. Then u is subharmonic in G if and only if*

$$A(u, x, r) \leqq M(u, x, r)$$

whenever $\bar{B}(x, r) \subset G$.

THEOREM B. *Let $u: G \to [0, \infty)$ be continuous in $G \subset \boldsymbol{R}^2$. Then u is* PL *if and only if*