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ON NORMAL SUBGROUPS OF CHEVALLEY GROUPS OVER COMMUTATIVE RINGS

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1. Introduction. Let G be an almost simple Chevalley-Demazure group scheme with root system Φ (see, for example [1], [2], [6], [7], [8], [10], [17], [19], [20], [21], [24]). For any commutative ring R with 1, let $E(R)$ denote the subgroup of $G(R)$ generated by all elementary unipotent (root elements) $x_\varphi(r)$ with φ in Φ and r in R. Here is an example: $G = SL_n$, $G(R) = SL_nR$, $E(R) = E_nR$, $\Phi = A_{n-1}$.

As in [1], [2], we are interested in normal subgroups of $G(R)$. More precisely, we want to describe all subgroups of $G(R)$ which are normalized by $E(R)$.

The case when the rank of G is 1, i.e. G is of type A_1, i.e. G is isogenous to $SL_2 = Sp_2$, is known to be exceptional (see, for example, [9]). So for the rest of this paper we assume that the rank of G is at least 2.

When R is a field, it is known [21] that every non-central subgroup of $G(R)$ normalized by $E(R)$ contains $E(R)$, unless G is of type C_2 or G_2 and R consists of two elements. In particular, with these exceptions, $E(R)$ modulo its center is a simple (abstract) group.

When R is not a field, there are normal subgroups of $G(R)$ involving (proper) ideals J of R. For every ideal J of R we define $G(R, J)$ to be the inverse image of the center of $G(R/J)$ under the canonical homomorphism $G(R) \to G(R/J)$. The kernel of this homomorphism, i.e. the congruence subgroup of level J, is denoted by $G(J)$. Let $E(J)$ denote the subgroup of $E(R) \cap G(J)$ generated by all $x_\varphi(u)$ with φ in Φ and u in J. Let $E(R, J)$ be the normal subgroup of $E(R)$ generated by $E(J)$.

THEOREM 1. *For any ideal J of R, the subgroup $E(R, J)$ of $G(R)$ is normal, and it contains the mixed commutator subgroup* $[E(R), G(J)]$.

When $G = SL_n$, Sp_{2n}, or SO_{2n}, this statement was proved: by Klingenberg [14, 15, 16] for local rings R; by Bass [4] and Bak [3] under stable range or similar dimensional conditions on R, by Suslin [22], Kopeiko [18], and Suslin-Kopeiko [23] for any commutative R.

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