Tôhoku Math. Journ.
38 (1986), 199-217.

STABLE MINIMAL SUBMANIFOLDS IN COMPACT RANK ONE SYMMETRIC SPACES

YOSHIHIRO OHNITA

(Received February 20, 1985)

Introduction. A compact submanifold M (without boundary) immersed in a Riemannian manifold $\bar{M}$ is called *minimal* if the first variation of its volume vanishes for every deformation of M in $\bar{M}$. Clearly, if the volume of M is a local minimum among all immersions, M is a minimal submanifold of $\bar{M}$. But the volume of a minimal submanifold is not always a local minimum. Nowadays we know a large number of examples of minimal submanifolds (e.g. totally geodesic submanifolds, complex submanifolds of Kaehler manifolds and extremal orbits of compact transformation groups, etc.). It is an important problem to know whether a given minimal submanifold has a local minimum volume or not.

We say that a compact minimal submanifold M in $\bar{M}$ is *stable* if the second variation of its volume is nonnegative for every deformation of M in $\bar{M}$. Clearly, if M has a local minimum volume, then it is stable. The class of stable minimal submanifolds is much smaller than the class of general minimal submanifolds. The existence of a stable minimal submanifold is closely related to the topological and Riemannian structures of the ambient manifold. In fact, Simons [13] and Lawson-Simons [9] proved the following remarkable theorems.

THEOREM A. *No p-dimensional compact minimal submanifold immersed in the Euclidean sphere S^n is stable for each p with $1 \leqq p \leqq n-1$.*

THEOREM B. *Let M be a p-dimensional compact minimal submanifold immersed in the complex projective space $P^n(\boldsymbol{C})$ with the Fubini-Study metric. Then M is stable if and only if $p = 2l$ for some integer $l \geqq 1$ and M is a complex submanifold in the sense that each tangent space of M is invariant under the complex structure of $P^n(\boldsymbol{C})$.*

The purpose of this paper is to complete the classification of compact stable minimal submanifolds in all compact rank one symmetric spaces (the sphere S^n, the real projective space $P^n(\boldsymbol{R})$, the complex projective space $P^n(\boldsymbol{C})$, the quaternionic projective space $P^n(\boldsymbol{H})$ and the Cayley projective plane $P^2(\boldsymbol{Cay})$). We will prove the following theorems.