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A CHARACTERIZATION OF THE STABLE INVARIANT INTEGRAL

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Contents

1. Introduction. Invariant integrals, stable or not, occupy a central position in harmonic analysis on a reductive Lie group G. For instance, they play a crucial role in Harish-Chandra's derivation of the Plancherel formula. They also figure prominently in the theory centering on the Selberg trace formula. Therefore it is only natural to try to characterize them. One important contribution in this direction is the work of Shelstad [9], who has obtained a "pointwise" description but only within the context of the Schwartz space $\mathscr{C}(G)$. For the applications, it is also necessary to consider other function spaces, e.g., $C_c^\infty(G)$. This, in fact, is one of our objectives. The main result is, however, rather different from Shelstad's in that the characterization is essentially "transform-theoretic" in nature (cf. [10]), the point being that the work of Herb [5-(b)] already gives explicit inversion formulae for the invariant integrals so, in order to study their transforms, a Paley-Wiener type theorem is required. And for this, the recent work of Clozel and Delorme [3-(a)] turns out to be exactly what is needed.

Regarding the organization, §§2–4 set up the preliminaries. In §5, we review the results of Herb and in §6 those of Clozel and Delorme. The characterization itself is the subject of §7. We close in §8 with a series of miscellaneous remarks that point the way to a number of variants on our main theme which can all be treated by the methods introduced here.

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