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OUTRADII OF THE TEICHMÜLLER SPACES OF FUCHSIAN GROUPS OF THE SECOND KIND

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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1. Introduction. Let $o(\Gamma)$ be the outradius of the Teichmüller space $T(\Gamma)$ of a Fuchsian group Γ. Then $o(\Gamma)$ is strictly greater than 2 (Earle [5]) and not greater than 6 (Nehari [7]). A Fuchsian group is said to be of the first kind (resp. second kind) if its region of discontinuity is not connected (resp. connected). If Γ is a finitely generated Fuchsian group of the first kind, then $o(\Gamma)$ is strictly less than 6 ([9]). Recently the authors proved, by using a basic result on the stability of finitely generated Fuchsian groups (Bers [3]), that $o(\Gamma)$ is equal to 6 for a finitely generated Fuchsian group Γ of the second kind ([10]). In this paper we give an alternative proof of it, which works also for an *infinitely generated* Fuchsian group of the second kind.

THEOREM. *If Γ is a Fuchsian group of the second kind, then $o(\Gamma)$ is equal to* 6.

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2. Definitions. Let Δ be the open unit disc and Δ^* be the exterior of Δ in the Riemann sphere $\hat{C}$. For each function f which is conformal in Δ^* let $\{f, z\}$ be the Schwarzian derivative of f, that is, $\{f, z\} = (f''/f')' - (1/2)(f''/f')^2$. Let Γ be a Fuchsian group keeping Δ invariant. A quasiconformal automorphism w of $\hat{C}$ is said to be compatible with Γ if $w\circ\gamma\circ w^{-1}$ is a Möbius transformation for each $\gamma\in\Gamma$. Let w be a quasiconformal automorphism of $\hat{C}$ which is compatible with Γ and which is conformal in Δ^*. The Teichmüller space $T(\Gamma)$ of Γ is the set of the Schwarzian derivatives $\{w|\Delta^*, z\}$ of such w's restricted to Δ^*. Let $\lambda(z) = (|z|^2 - 1)^{-1}$ be a Poincaré density of Δ^*. For a function ϕ defined in Δ^* let $\|\phi\| = \sup_{z\in\Delta^*}\lambda(z)^{-2}|\phi(z)|$. The outradius $o(\Gamma)$ of $T(\Gamma)$ is defined to be $\sup\|\phi\|$, where the supremum is taken over all ϕ in $T(\Gamma)$.

3. Lemmas. In this section we state two lemmas without proof.