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TOTALLY GEODESIC FOLIATIONS AND KILLING FIELDS, II

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1. Introduction. A foliation $\mathscr{F}$ of a Riemannian manifold (M, g) is said to be totally geodesic if every leaf of $\mathscr{F}$ is a totally geodesic submanifold of (M, g). In [6], Johnson and Whitt studied some properties of Killing fields on complete connected Riemannian manifolds admitting codimension-one totally geodesic foliations by compact leaves. In [7], the author studied one of these properties of Killing fields on closed Riemannian manifolds admitting not necessarily compact codimension-one totally geodesic foliations and proved the following: Let (M, g) be a closed connected Riemannian manifold and $\mathscr{F}$ be a codimension-one totally geodesic foliation of (M, g). Then any Killing field Z on (M, g) preserves $\mathscr{F}$, that is, the flow of Z maps each leaf of $\mathscr{F}$ to a leaf of $\mathscr{F}$.

In this paper, we extend this result to higher codimensions by studying Jacobi fields along geodesics on totally geodesic leaves. We prove the following.

THEOREM. *Let (M, g) be a connected complete Riemannian manifold and $\mathscr{F}$ be a totally geodesic foliation of (M, g). Assume that the bundle orthogonally complement to $\mathscr{F}$ is also integrable. Then any Killing field Z on (M, g) with bounded length, i.e., $g(Z, Z) \leqq \text{const.} < \infty$ on M, preserves $\mathscr{F}$.*

The proof will be given in Section 3. In Section 4, we give some examples and study a related topic.

2. Preliminaries. Let (M, g) be a connected complete Riemannian manifold and $\mathscr{F}$ be a codimension-q totally geodesic foliation of (M, g). Denote by D the Riemannian connection of (M, g) and by R the curvature tensor of D. We also denote $g(X, Y)$ by $\langle X, Y\rangle$. Let $c: \boldsymbol{R} \to M$ be a geodesic parametrized by arc length on a totally geodesic leaf L of $\mathscr{F}$ and $Y(t)$ be a Jacobi field along c. Then $Y(t)$ satisfies the Jacobi equation $D_{c'(t)}D_{c'(t)}Y(t) + R_tY(t) = 0$ where $R_tY(t) = R(Y(t), c'(t))c'(t)$. Set $x = c(0)$. We choose an orthonormal basis $\{E_1, \cdots, E_p, X_1, \cdots, X_q\}$ of T_xM with $E_1 = c'(0)$, $E_2, \cdots, E_p \in T_x\mathscr{F}$ and $X_1, \cdots, X_q \in T_x\mathscr{F}^{\perp}$ where $\dim(L) = p$