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ON THE GROWTH OF MEROMORPHIC SOLUTIONS OF SOME ALGEBRAIC DIFFERENTIAL EQUATIONS

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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1. Introduction. In this paper we shall study the growth of meromorphic solutions of some algebraic differential equations with the aid of the Nevanlinna theory of meromorphic functions (see [4], [6]). We denote by M the set of meromorphic functions in the complex plane, by E some subset of $[0, \infty)$ with $\operatorname{meas} E < \infty$ and by K some constant which is not always the same. The term "meromorphic" will mean meromorphic in the complex plane.

Let H be a differential polynomial of $w, w', \cdots, w^{(\mu)}$ $(\mu \geqq 1)$ with coefficients in M:

$$H = H(w, w', \cdots, w^{(\mu)}) = \sum_{\lambda \in I} c_\lambda(z) w^{q_0}(w')^{q_1} \cdots (w^{(\mu)})^{q_\mu} ,$$

where $c_\lambda \in M$ with $c_\lambda \neq 0$ and where I is a finite set of multi-indices $\lambda = (q_0, q_1, \cdots, q_\mu)$ of nonnegative integers $q_0, q_1, \cdots, q_\mu$. Let $Q_i(w)$ be a polynomial in w with coefficients in M:

$$Q_i = Q_i(w) = \sum_{j=0}^{m_i} a_{ij} w^j \quad (a_{ij} \in M,\ i = 0, 1, \cdots, n) .$$

Consider the differential equation (D.E., for short):

$$(1) \qquad F(w, H) = Q_n(w)H^n + \cdots + Q_1(w)H + Q_0(w) = 0 ,$$

where $Q_n(w) \neq 0$ and $F(w, H)$ is irreducible over M as a polynomial in w and H. A meromorphic solution $w = w(z)$ is said to be admissible if

$$T(r, f) = o(T(r, w)) \quad (r \to \infty,\ r \notin E)$$

for all coefficients $f = a_{ij}$, c_λ in (1).

Eremenko [1] gave the following:

"Suppose that the D.E. (1) has an admissible solution. Then,

(i) $m_n = 0$;

(ii) When $H = w^{(\mu)}$,

$$(2) \qquad m_j \leqq (\mu + 1)(n - j) \quad (j = 0, 1, \cdots, n) ."$$